How to Find Determinants of 3x3 Matrices

Way #1: Expansion by Minors

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a\begin{vmatrix} e & f \\ h & i \end{vmatrix} - b\begin{vmatrix} d & f \\ g & i \end{vmatrix} + c\begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

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Example: Find the determinant using expansion by minors.

$$\begin{vmatrix} 3 & -1 & 4 \\ 2 & -2 & 5 \\ 4 & -1 & 0 \end{vmatrix}$$

$$3\begin{vmatrix} -2 & 5 \\ -1 & 0 \end{vmatrix} - 1\begin{vmatrix} 2 & 5 \\ 4 & 0 \end{vmatrix} + 4\begin{vmatrix} 2 & -2 \\ 4 & -1 \end{vmatrix}$$

$$3(0 - -5) + 1(0 - 20) + 4(-2 - -8)$$

$$3(5) + 1(-20) + 4(6)$$

$$15 + -20 + 24 = 19$$

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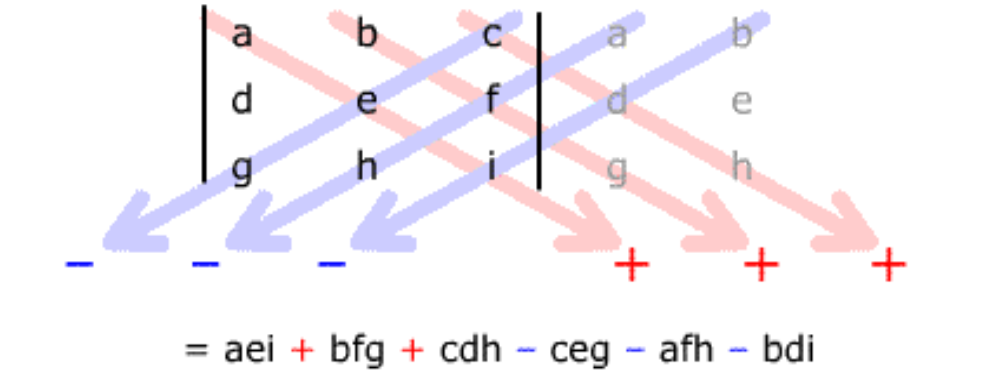
Way #2: Diagonals



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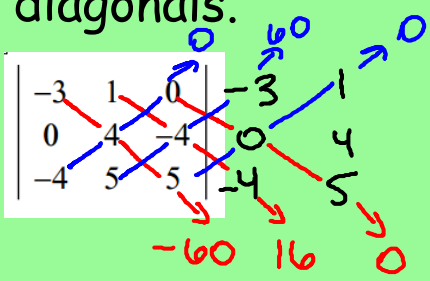
Example: Find the determinant using diagonals.



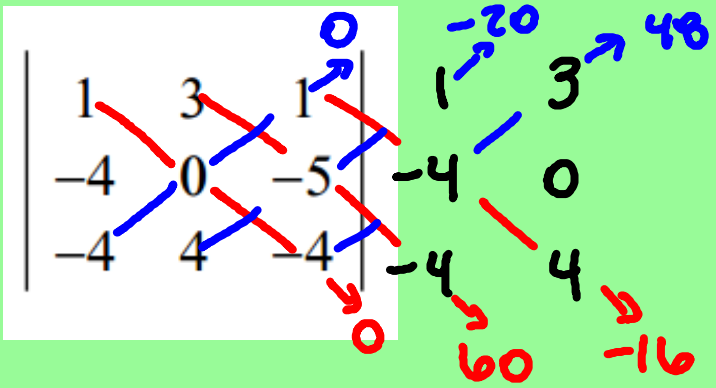
$-60+16+0-0-60-0$

$-60+16-60$

-104

Example: Find the determinant.

$$\begin{vmatrix} 1 & 3 & 1 \\ -4 & 0 & -5 \\ -4 & 4 & -4 \end{vmatrix}$$



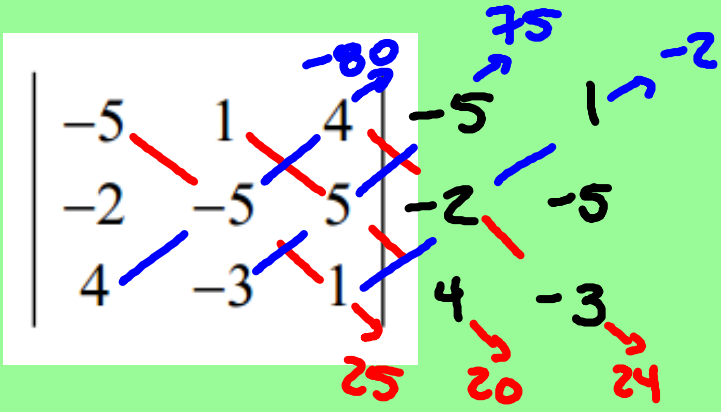
$0+60+-16-0--20-48$

$60-16+20-48$

16

Example: Find the determinant.

$$\begin{vmatrix} -5 & 1 & 4 \\ -2 & -5 & 5 \\ 4 & -3 & 1 \end{vmatrix}$$



$25+20+24++80-75++2$

76

Example: Find the determinant.

$$\begin{vmatrix} -1 & -3 & 3 \\ -5 & 2 & -5 \\ 5 & 0 & -1 \end{vmatrix}$$

$$-1\begin{vmatrix} 2 & -5 \\ 0 & -1 \end{vmatrix} - -3\begin{vmatrix} -5 & -5 \\ 5 & -1 \end{vmatrix} + 3\begin{vmatrix} -5 & 4 \\ 5 & 0 \end{vmatrix}$$

$$-1(-2-0) + 3(5 + +25) + 3(0-10)$$

$$-1(-2) + 3(30) + 3(-10)$$

$$2 + 90 + -30 = 62$$

Example: Find the determinant.

$$\begin{vmatrix} 4 & 5 & 1 \\ 3 & -3 & -5 \\ -5 & 3 & -3 \end{vmatrix}$$

$$4\begin{vmatrix} -3 & -5 \\ 3 & -3 \end{vmatrix} - 5\begin{vmatrix} 3 & -5 \\ -5 & -3 \end{vmatrix} + 1\begin{vmatrix} 3 & -3 \\ -5 & 3 \end{vmatrix}$$

$$4(9 - -15) - 5(-9-25) + 1(9-15)$$

$$4(24) - 5(-34) + 1(-6)$$

$$96 + 170 + -6 = 260$$