

## 3.1 Fractions &amp; Decimals

Some fractions like  $\frac{1}{2}$  and  $\frac{3}{4}$  can be written as a decimal by making equivalent fractions with denominators of 10, 100, or 1000. However, any fraction  $\frac{a}{b}$ , where  $b \neq 0$ , can be written as a decimal by dividing the numerator by the denominator.

$$\frac{a}{b} = a \div b$$

The decimal form of a rational number is called a repeating decimal. If the repeating digit is zero, then the decimal is a terminating decimal.

**Example:** Write each fraction as a decimal.

a.)  $\frac{7}{8} = 7 \div 8$

$$\begin{array}{r} \boxed{0.875} \\ 8 \overline{) 7.000} \\ \underline{-64} \phantom{00} \\ 60 \phantom{0} \\ \underline{-56} \phantom{0} \\ 40 \\ \underline{-40} \\ 0 \end{array}$$

b.)  $\frac{4}{5}$

$$\begin{array}{r} \boxed{0.8} \\ 5 \overline{) 4.0} \\ \underline{-40} \\ 0 \end{array}$$

c.)  $\frac{3}{16}$

$$\begin{array}{r} \boxed{0.1875} \\ 16 \overline{) 3.0000} \\ \underline{-16} \phantom{000} \\ 140 \phantom{0} \\ \underline{-128} \phantom{0} \\ 120 \phantom{0} \\ \underline{-112} \phantom{0} \\ 80 \\ \underline{-80} \\ 0 \end{array}$$

Not all fractions have repeating digits that are zero. Sometimes a nonzero digit of a group of digits repeats without end in the quotient.

$$\frac{4}{9} \longrightarrow \begin{array}{r} .444\dots \\ 9 \overline{) 4.000} \\ \underline{-36} \phantom{0} \\ 40 \phantom{0} \\ \underline{-36} \phantom{0} \\ 40 \phantom{0} \\ \underline{-36} \\ 4 \end{array} \quad 0.\overline{4}$$

You can indicate that the digit 4 repeats by annexing dots.

So,  $\frac{4}{9} = 0.44444444\dots$ . This decimal is called a repeating decimal. Repeating decimals have a pattern in their digits that repeats without end. Bar notation is a bar or line placed over the digit(s) that repeats.

**Example:** Write each fraction as a decimal. Use a bar to show a repeating decimal.

d.)  $\frac{5}{12} = 0.4\overline{16}$

$$\begin{array}{r} 41\overline{66} \\ 12 \overline{) 5.00000} \\ \underline{-48} \phantom{000} \\ 20 \phantom{00} \\ \underline{-12} \phantom{00} \\ 80 \phantom{0} \\ \underline{-72} \phantom{0} \\ 80 \phantom{0} \\ \underline{-72} \phantom{0} \\ 8 \dots \end{array}$$

e.)  $\frac{-2}{11} = -0.\overline{18}$

$$\begin{array}{r} 18\overline{18} \\ 11 \overline{) 2.00000} \\ \underline{-11} \phantom{0000} \\ 90 \phantom{00} \\ \underline{-88} \phantom{00} \\ 20 \phantom{0} \\ \underline{-18} \phantom{0} \\ 20 \phantom{0} \\ \underline{-18} \phantom{0} \\ 2 \dots \end{array}$$

**Example:** In a recent Masters Tournament, Zach Johnson's first shot landed on the fairway 45 out of 56 times. To the nearest thousandth, what part of the time did his shot land on the fairway?

3 decimal points

$$\begin{array}{r} 5 \\ 56 \\ \times 9 \\ \hline 504 \end{array}$$

$$\begin{array}{r} 45 \\ \hline 56 \end{array}$$

$$\begin{array}{r} 4 \\ 56 \\ \times 8 \\ \hline 448 \end{array}$$

$$\begin{array}{r} 4 \\ 56 \\ \times 7 \\ \hline 392 \end{array}$$

$$\begin{array}{r} 3 \\ 56 \\ \times 6 \\ \hline 336 \end{array}$$

$$\begin{array}{r} 3 \\ 56 \\ \times 5 \\ \hline 280 \end{array}$$

$$\begin{array}{r} 4 \\ 56 \\ \times 4 \\ \hline 224 \end{array}$$

$$\begin{array}{r} 1 \\ 56 \\ \times 3 \\ \hline 168 \end{array}$$

$$\begin{array}{r} 20 \overline{) 45.0000} \\ \underline{-44} \phantom{0000} \\ 100 \phantom{00} \\ \underline{-80} \phantom{00} \\ 200 \phantom{0} \\ \underline{-168} \phantom{0} \\ 320 \\ \underline{-280} \\ 40 \end{array}$$

$$0.8035$$

$$\boxed{0.804}$$

**Example:** Replace each space with <, >, or = to make a true sentence.

f.)  $\frac{1}{4} > 0.2$

$$\begin{array}{r} 25 \\ 4 \overline{) 1.00} \\ \underline{-8} \phantom{00} \\ 20 \\ \underline{-20} \\ 0 \end{array}$$

g.)  $\frac{-7}{15} < \frac{-5}{12}$

$$\begin{array}{r} 466 \\ 15 \overline{) 7.000} \\ \underline{-60} \phantom{00} \\ 100 \phantom{0} \\ \underline{-90} \phantom{0} \\ 100 \\ \underline{-90} \\ 10 \end{array}$$

$$\begin{array}{r} 4166 \\ 12 \overline{) 5.0000} \\ \underline{-48} \phantom{0000} \\ 180 \phantom{00} \\ \underline{-120} \phantom{00} \\ 600 \phantom{0} \\ \underline{-600} \\ 0 \end{array}$$