

3x3 Inverse Matrices

To find the inverse of a 3×3 matrix,

1. Compute the minors of each element
2. Negate every other element, according to a checkerboard pattern
3. Take the transpose
4. Divide by the determinant of the original matrix

Pictorially, this can be represented as:

$$\begin{pmatrix} x & x & x \\ x & x & x \\ x & x & x \end{pmatrix}^{-1} : \begin{pmatrix} \text{minors} \end{pmatrix} \rightarrow \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix} \rightarrow \begin{pmatrix} \text{transpose} \end{pmatrix} \rightarrow \frac{1}{\det}$$

The minor of the (i, j) th entry of a matrix \mathbf{A} is the determinant of the submatrix obtained by removing the i th row and the j th column of \mathbf{A} .

Example: Find the inverse of the matrix.

$$\begin{bmatrix} -1 & 4 & 5 \\ 3 & 6 & -2 \\ 4 & 3 & 1 \end{bmatrix}$$

$$\det = -131$$

$$\begin{bmatrix} +12 & -11 & +15 \\ -11 & +21 & -19 \\ +38 & -13 & +18 \end{bmatrix} = \begin{bmatrix} 12 & -11 & -15 \\ 11 & -21 & 19 \\ -38 & 13 & -18 \end{bmatrix} = \frac{1}{-131} \begin{bmatrix} 12 & 11 & -38 \\ -11 & -21 & 13 \\ -15 & 19 & -18 \end{bmatrix}$$

Example: Find the inverse of the matrix.

$$\begin{bmatrix} 3 & -2 & -3 \\ 4 & 2 & -8 \\ -4 & 1 & 5 \end{bmatrix}$$

det = -6

$$\begin{bmatrix} \underline{+18} & \underline{-12} & \underline{+12} \\ \underline{-7} & \underline{+3} & \underline{-5} \\ \underline{+22} & \underline{-12} & \underline{+14} \end{bmatrix} = \begin{bmatrix} 18 & 12 & 12 \\ 7 & 3 & 5 \\ 22 & 12 & 14 \end{bmatrix} = \frac{1}{-6} \begin{bmatrix} 18 & 7 & 22 \\ 12 & 3 & 12 \\ 12 & 5 & 14 \end{bmatrix}$$

Example: Find the inverse of the matrix.

$$\begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -1 \\ -1 & 1 & 2 \end{bmatrix}$$

$\begin{matrix} -1 \rightarrow 0 \\ 2 \rightarrow 1 \\ -1 \rightarrow 1 \end{matrix}$
 $\begin{matrix} 0 \rightarrow 0 \\ 1 \\ 1 \end{matrix}$
 $\begin{matrix} -2+0+2 = -1-1-0 \\ 0+0 \end{matrix}$

 det = 0

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Attachments



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