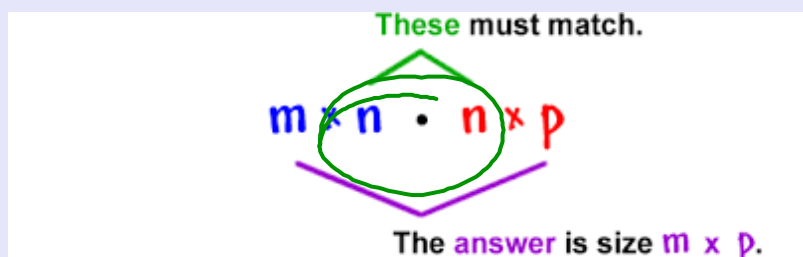


4.2 Multiplication of Matrices

In order to multiply matrices, the number of columns in the first matrix must equal the number of rows in the second.



Multiplying Matrices

Use the i^{th} row of the first matrix and the j^{th} column of the second matrix. Multiply the corresponding elements and add the products.

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e & f \\ g & h \end{pmatrix} = \begin{pmatrix} (ae + bg) & (af + bh) \\ (ce + dg) & (cf + dh) \end{pmatrix}$$

Example: Perform the indicated operation.

$$2 \times 2 \checkmark 2 \times 2 \Rightarrow 2 \times 2$$

$$\begin{bmatrix} 0 & 2 \\ -2 & -5 \end{bmatrix} \cdot \begin{bmatrix} 6 & -6 \\ 3 & 0 \end{bmatrix}$$

$$\begin{bmatrix} \underline{0 \cdot 6 + 2 \cdot 3} & \underline{0 \cdot (-6) + 2 \cdot 0} \\ \underline{-2 \cdot 6 + (-5) \cdot 3} & \underline{-2 \cdot (-6) + (-5) \cdot 0} \end{bmatrix} = \begin{bmatrix} 0+6 & 0+0 \\ -12+(-15) & 12+0 \end{bmatrix} = \begin{bmatrix} 6 & 0 \\ -27 & 12 \end{bmatrix}$$

Example: Perform the indicated operation.

$$2 \times 1 \checkmark 1 \times 2 \Rightarrow 2 \times 2$$

$$\begin{bmatrix} 6 \\ -3 \end{bmatrix} \cdot \begin{bmatrix} -5 & 4 \end{bmatrix}$$

$$\begin{bmatrix} \underline{6 \cdot (-5)} & \underline{6 \cdot 4} \\ \underline{-3 \cdot (-5)} & \underline{-3 \cdot 4} \end{bmatrix} = \begin{bmatrix} -30 & 24 \\ 15 & -12 \end{bmatrix}$$

Example: Perform the indicated operation.

$$2 \times 2 \checkmark 2 \times 2 = 2 \times 2$$

$$\begin{bmatrix} -3 & 5 \\ -2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 6 & -2 \\ 1 & -5 \end{bmatrix}$$

$$\begin{bmatrix} -3 \cdot 6 + 5 \cdot 1 & -3 \cdot -2 + 5 \cdot -5 \\ -2 \cdot 6 + 1 \cdot 1 & -2 \cdot -2 + 1 \cdot -5 \end{bmatrix} = \begin{bmatrix} -18 + 5 & 6 + -25 \\ -12 + 1 & 4 + -5 \end{bmatrix} = \begin{bmatrix} -13 & -19 \\ -11 & -1 \end{bmatrix}$$

Example: Perform the indicated operation.

$$2 \times 3 \checkmark 3 \times 2 \Rightarrow 2 \times 2$$

$$\begin{bmatrix} 5 & 3 & 5 \\ 1 & 5 & 0 \end{bmatrix} \cdot \begin{bmatrix} -4 & 2 \\ -3 & 4 \\ 3 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 5 \cdot -4 + 3 \cdot -3 + 5 \cdot 3 & 5 \cdot 2 + 3 \cdot 4 + 5 \cdot -5 \\ 1 \cdot -4 + 5 \cdot -3 + 0 \cdot 3 & 1 \cdot 2 + 5 \cdot 4 + 0 \cdot -5 \end{bmatrix} = \begin{bmatrix} -20 + -9 + 15 & 10 + 12 + -25 \\ -4 + -15 + 0 & 2 + 20 + 0 \end{bmatrix} = \begin{bmatrix} -14 & -3 \\ -19 & 22 \end{bmatrix}$$

Example: Perform the indicated operation.

$$\begin{matrix} 2 \times 3 & \times & 2 \times 3 \\ \begin{bmatrix} 3 & 2 & 5 \\ 2 & 3 & 1 \end{bmatrix} & \cdot & \begin{bmatrix} 4 & 5 & -5 \\ 5 & -1 & 6 \end{bmatrix} \end{matrix}$$

IMPOSSIBLE

Example: Perform the indicated operation.

$$\begin{matrix} 2 \times 2 & \cdot & 2 \times 1 = 2 \times 1 \\ \begin{bmatrix} 5 & 4 \\ 2 & -1 \end{bmatrix} & \cdot & \begin{bmatrix} -4 \\ 3 \end{bmatrix} \end{matrix}$$

$$\begin{bmatrix} 5 \cdot 4 + 4 \cdot 3 \\ 2 \cdot 4 + (-1) \cdot 3 \end{bmatrix} = \begin{bmatrix} -20 + 12 \\ -8 + 3 \end{bmatrix} = \begin{bmatrix} -8 \\ -11 \end{bmatrix}$$

Example: Perform the indicated operation.

$$\begin{array}{c}
 4 \times 3 \quad \checkmark \quad 3 \times 2 = 4 \times 2 \\
 \begin{bmatrix} -1 & 1 & -1 \\ 5 & 2 & -5 \\ 6 & -5 & 1 \\ -5 & 6 & 0 \end{bmatrix} \cdot \begin{bmatrix} 6 & 5 \\ 5 & -6 \\ 6 & 0 \end{bmatrix}
 \end{array}$$

$$\begin{array}{l}
 \underline{-6+5+-6} \quad \underline{-5+-6+0} \\
 \underline{30+10+-30} \quad \underline{25+-12+0} \\
 \underline{36+-25+6} \quad \underline{30+30+0} \\
 \underline{-30+30+0} \quad \underline{-25+-36+0}
 \end{array}
 = \begin{bmatrix} -7 & -11 \\ 10 & 13 \\ 17 & 60 \\ 0 & -61 \end{bmatrix}$$

Example: Perform the indicated operation.

$$\begin{array}{c}
 1 \times 2 \quad \checkmark \quad 2 \times 2 = 1 \times 2 \\
 \begin{bmatrix} 2 & -5v \end{bmatrix} \cdot \begin{bmatrix} -5u & -v \\ 0 & 6 \end{bmatrix}
 \end{array}$$

$$\begin{bmatrix} \underline{-10u+0} & \underline{-2v+-30v} \end{bmatrix} = \begin{bmatrix} -10u & -32v \end{bmatrix}$$