4.1 Matrix Operations

A matrix is a system of rows and columns. A matrix is a problem-solving tool that organizes numbers or data so that each position in the matrix has a purpose.

In algebra, a matrix is not expressed as a table, but as an array of values. Each value is called an element of the matrix.

A matrix is named by using the matrix dimensions with the letter name. The dimensions tell how many rows and columns there are in the matrix.

row × column

$A_{2\times 3}$

$$A = \underbrace{\begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix}}_{\text{3 columns}} \Big\} \text{2 rows}$$

Some *matrices* (the plural of *matrix*) have special names because of their dimensions or entries.

NAME	DESCRIPTION	EXAMPLE
Row matrix	A matrix with only 1 row	$\begin{bmatrix} 3 & -2 & 0 & 4 \end{bmatrix}$
Column matrix	A matrix with only 1 column	$\begin{bmatrix} 1 \\ 3 \end{bmatrix}$
Square matrix	A matrix with the same number of rows and columns	$\begin{bmatrix} 4 & -1 & 5 \\ 2 & 0 & 1 \\ 1 & -3 & 6 \end{bmatrix}$
Zero matrix	A matrix whose entries are all zeros	$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

Two matrices are equal if their dimensions are the same and each element in the corresponding positions are equal.

COMPARING MATRICES Tell whether the matrices are *equal* or *not equal*.

11. $\underset{1\times3}{\begin{bmatrix} 5 & -1 & 7 \end{bmatrix}}, \underset{3\times1}{\begin{bmatrix} 5 \\ -1 \\ 7 \end{bmatrix}}$ not equal

12. $\underset{2\times3}{\begin{bmatrix} 1 & 0 & -8 \\ 8 & 0 & 1 \end{bmatrix}}, \underset{2\times3}{\begin{bmatrix} 1 & 0 & -8 \\ 8 & 0 & 1 \end{bmatrix}}$ EQUAL

13. $\underset{2\times2}{\begin{bmatrix} 4 & 0 \\ 2 & -4 \end{bmatrix}}, \underset{2\times2}{\begin{bmatrix} 4 & 0 \\ -2 & -4 \end{bmatrix}}$ not equal

14. $\underset{2\times3}{\begin{bmatrix} 2 & 1.5 & 4.25 \\ 0.5 & -0.5 & 0 \end{bmatrix}}, \underset{2\times3}{\begin{bmatrix} 2 & \frac{3}{2} & \frac{17}{4} \\ \frac{1}{2} & -\frac{1}{2} & 0 \end{bmatrix}}$ EQUAL

Using the definition of equal matrices, we can find values when elements of the matrices are algebraic expressions.

EXAMPLE

$$\underset{2\times1}{\begin{bmatrix} 2x + y \\ x - 3y \end{bmatrix}} = \underset{2\times1}{\begin{bmatrix} 6 \\ 31 \end{bmatrix}}$$

$x - 3y = 31$
$+3y \quad +3y$
$x = 31 + 3y$

$2x + y = 6$
$2(31 + 3y) + y = 6$
$62 + 6y + y = 6$
$62 + 7y = 6$
$-62 \qquad -62$
$\frac{7y}{7} = \frac{-56}{7}$
$\boxed{y = -8}$

$x = 31 + 3(-8)$
$x = 31 + -24$
$\boxed{x = 7}$

You can multiply any matrix by a constant. This is called scalar multiplication. When scalar multiplication is performed, each element is multiplied by the constant, and a new matrix is formed.

$$m\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ma & mb & mc \\ md & me & mf \end{bmatrix}$$

Example: Perform the indicated operation.

$$-4\begin{bmatrix} -1 & -3 & 7 \\ -4 & 0 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 12 & -28 \\ 16 & 0 & -24 \end{bmatrix}$$

$$4\begin{bmatrix} 1 & 3 & 9 \\ -5 & 5 & 15 \\ -3 & -5 & -11 \end{bmatrix}$$

$$\begin{bmatrix} 4 & 12 & 36 \\ -20 & 20 & 60 \\ -12 & -20 & -44 \end{bmatrix}$$

Matrices can also be added and subtracted. In order to do so, they must have the same dimensions. Then, add or subtract the corresponding elements.

If $A = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$, **and** $C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$, **find each sum. If the sum does not exist, write** ***impossible*****.**

a. $A + B$

$$A + B = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} \quad \text{Substitution}$$

$$= \begin{bmatrix} 3+7 & -4+(-4) & 7+(-2) \\ -1+1 & 6+6 & 0+(-3) \end{bmatrix} \quad \text{Definition of matrix addition}$$

$$= \begin{bmatrix} 10 & -8 & 5 \\ 0 & 12 & -3 \end{bmatrix} \quad \text{Simplify.}$$

b. $B + C$

$$B + C = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix} \quad \text{Substitution}$$

Since B is a 2-by-3 matrix and C is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.

<u>Example:</u> Perform the indicated operation on the matrices.

a. $\underset{3\times 1}{\begin{bmatrix} 3 \\ -4 \\ 7 \end{bmatrix}} + \underset{3\times 1}{\begin{bmatrix} 1 \\ 0 \\ 3 \end{bmatrix}}$

$$\begin{bmatrix} 4 \\ -4 \\ 10 \end{bmatrix}$$

b. $\underset{2\times 2}{\begin{bmatrix} 8 & 3 \\ 4 & 0 \end{bmatrix}} - \underset{2\times 2}{\begin{bmatrix} 2 & -7 \\ 6 & -1 \end{bmatrix}}$

$$\begin{bmatrix} 6 & 10 \\ -2 & 1 \end{bmatrix}$$

c. $\underset{2\times 2}{\begin{bmatrix} 2 & 0 \\ 3 & 4 \end{bmatrix}} + \underset{2\times 1}{\begin{bmatrix} 1 \\ 5 \end{bmatrix}}$

<u>IMPOSSIBLE</u>

Example: Perform the indicated operation on the matrices.

$$\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + 4\begin{bmatrix} -1 & 0 \\ 3 & -2 \\ -4 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 12 & -8 \\ -16 & 20 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & -3 \\ -16 & 23 \end{bmatrix}$$

Example: Perform the indicated operation on the matrices.

$$3\begin{bmatrix} -7 & 1 & 0 \\ 8 & -6 & -2 \end{bmatrix} - 2\begin{bmatrix} 4 & -1 & -7 \\ -3 & -5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} -21 & 3 & 0 \\ 24 & -18 & -6 \end{bmatrix} + \begin{bmatrix} -8 & 2 & 14 \\ 6 & 10 & -10 \end{bmatrix}$$

$$\begin{bmatrix} -29 & 5 & 14 \\ 30 & -8 & -16 \end{bmatrix}$$