Column

4.1

Matrix Operations

A matrix is a system of rows and columns. A matrix is a problem-solving tool that organizes numbers or data so that each position in the matrix has a purpose.

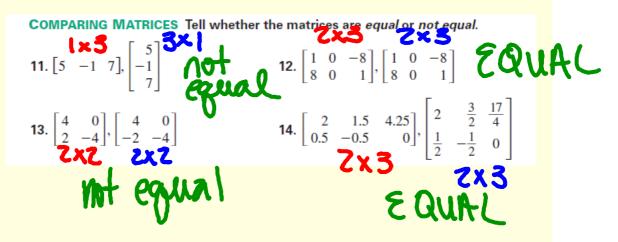
In algebra, a matrix is not expressed as a table, but as an array of values. Each value is called an element of the matrix.

A matrix is named by using the matrix dimensions with the letter name. The dimensions tell how many rows and columns there are in

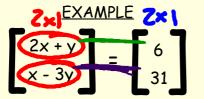
	the matrix.	row	×
Azxa	$A = \begin{bmatrix} 6 & 2 & -1 \\ -2 & 0 & 5 \end{bmatrix} $ 2 roo		
	3 columns		
Some <i>matrices</i> (the plural of <i>matrix</i>) have special names because of their dimensions or entries.			
NAME	DESCRIPTION	EXAMPLE	
Row matrix	A matrix with only 1 row	[3 -2 0 4]	
Column matrix	A matrix with only 1 column	$\begin{bmatrix} 1\\ 3 \end{bmatrix}$	
Square matrix	A matrix with the same number	$\begin{bmatrix} 4 & -1 & 5 \\ 2 & 0 & 1 \end{bmatrix}$	

 $\begin{array}{cccc}
 2 & 0 & 1 \\
 1 & -3 & 6
 \end{array}$ of rows and columns 0 0] 0 0 A matrix whose entries are all zeros Zero matrix 0 0

Two matrices are equal if their dimensions are the same and each element in the corresponding positions are equal.



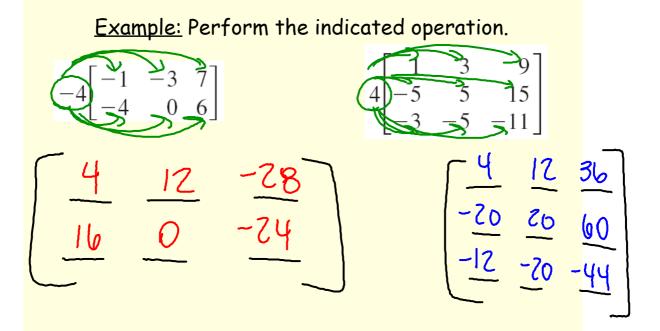
Using the definition of equal matrices, we can find values when elements of the matrices are algebraic expressions.



$$Z = - 5b$$

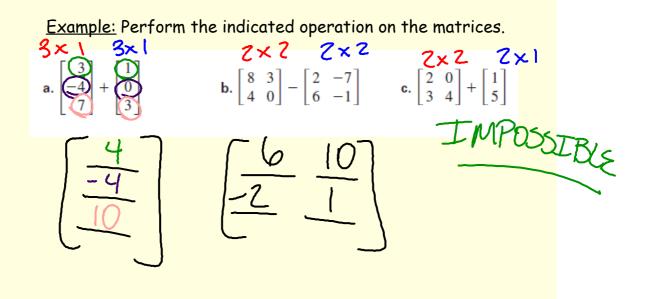
You can multiply any matrix by a constant. This is called scalar multiplication. When scalar multiplication is performed, each element is multiplied by the constant, and a new matrix is formed.

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} = \begin{bmatrix} ma & mb & mc \\ md & me & mf \end{bmatrix}$$



Matrices can also be added and subtracted. In order to do so, they must have the same dimensions. Then, add or subtract the corresponding elements. If $A = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$, and $C = \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$, find each sum. If the sum does not exist, write *impossible*. a. A + B $A + B = \begin{bmatrix} 3 & -4 & 7 \\ -1 & 6 & 0 \end{bmatrix} + \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix}$ Substitution $= \begin{bmatrix} 3+7 & -4+(-4) & 7+(-2) \\ -1+1 & 6+6 & 0+(-3) \end{bmatrix}$ Definition of matrix addition $= \begin{bmatrix} 10 & -8 & 5 \\ 0 & 12 & -3 \end{bmatrix}$ Simplify. b. B + C $B + C = \begin{bmatrix} 7 & -4 & -2 \\ 1 & 6 & -3 \end{bmatrix} + \begin{bmatrix} 3 & 6 \\ -4 & 5 \end{bmatrix}$ Substitution

Since *B* is a 2-by-3 matrix and *C* is a 2-by-2 matrix, the matrices do not have the same dimensions. Therefore, it is impossible to add these matrices.



Example: Perform the indicated operation on the matrices.

$$\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + 4 \begin{bmatrix} -1 & 0 \\ 3 & -2 \\ -4 & 5 \end{bmatrix}$$
$$\begin{bmatrix} 12 & -8 \\ 0 & 5 \\ 0 & 5 \\ 0 & 3 \end{bmatrix} + \begin{bmatrix} -4 & 0 \\ 12 & -8 \\ -14 & 20 \end{bmatrix} = \begin{bmatrix} 8 & -8 \\ 12 & -3 \\ -16 & 73 \end{bmatrix}$$

Example: Perform the indicated operation on the matrices.

$$\begin{bmatrix} -7 & 1 & 0 \\ 3 & -7 & 1 & 0 \\ -7 & -6 & -2 \end{bmatrix} \xrightarrow{4} \xrightarrow{4} \xrightarrow{-1} \xrightarrow{-7} \xrightarrow{-7} \xrightarrow{-6} \xrightarrow{-2} \xrightarrow{-6} \xrightarrow{-2} \xrightarrow{-6} \xrightarrow{-5} \xrightarrow{-5} \xrightarrow{-5} \xrightarrow{-10} \xrightarrow{-29} \xrightarrow{-7} \xrightarrow{-10} \xrightarrow{-7} \xrightarrow{-7$$