

SOLVING RATIONAL EQUATIONS

To solve a rational equation,
 * multiply each term on both sides
 of the equation by the LCD.*

*gets rid
of
fractions*

Example 1: Solve. $\frac{4}{x} + \frac{5}{2} = -\frac{11}{x}$

LCD: $2x$

$$\begin{array}{r} 8 + 5x = -22 \\ -8 \qquad -8 \end{array}$$

* Check for
extraneous
solutions..

We cannot have

0 in the denominator!

$$\frac{5x}{5} = \frac{-30}{5}$$

$$x = -6$$

Example 2: Solve. $\frac{3}{x} - \frac{1}{2} = \frac{12}{x}$

LCD: $2x$

$$\begin{array}{r} 6 - x = 24 \\ -6 \qquad -6 \end{array}$$

$$\frac{-x}{-1} = \frac{18}{-1}$$

$$\boxed{x = -18}$$

Example 3: Solve.

$$\frac{5x}{x-2} = 7 + \frac{10}{x-2}$$

(Handwritten notes: $(x-2)$ above the first fraction, $(x-2)$ above the second fraction, $x-2$ circled in red under both fractions, and $2-2=0$ written in red below each denominator.)

LCD: $x-2$

$$5x = 7x - 14 + 10$$

$$5x = 7x - 4$$

$$-7x \quad -7x$$

$$\frac{-2x}{-2} = \frac{-4}{-2}$$

$$\cancel{x=2}$$

NO
SOLUTION

Example 4: Solve:

$$\frac{5x}{x+1} = 4 - \frac{5}{x+1}$$

(Handwritten notes: Red arrows point from $(x+1)$ to the denominators. Below the denominators, $-1+1=0$ is written.)

LCD: $x+1$

$$5x = 4x + 4 - 5$$

$$5x = 4x - 1$$

(Handwritten notes: $-4x$ is written below both $5x$ and $4x$.)

$$\cancel{x = -1}$$

NO SOLUTION

Example 5: Solve: $\frac{4x+1}{x+1} = \frac{12}{x^2-1} + 3$

LCD: $(x+1)(x-1)$

$$(x-1)(4x+1) = 12 + 3(x+1)(x-1)$$

$$4x^2 + x - 4x - 1 = 12 + 3(x^2 - x + x - 1)$$

$$4x^2 - 3x - 1 = 12 + 3(x^2 - 1)$$

$$4x^2 - 3x - 1 = 12 + 3x^2 - 3$$

$$4x^2 - 3x - 1 = 3x^2 + 9$$

$$x^2 - 3x - 1 = 9$$

$$\begin{array}{r|l} s-3 & p-10 \\ \hline -5+2 & -5 \cdot 2 \end{array}$$

$$x^2 - 3x - 10 = 0$$

$$(x-5)(x+2) = 0$$

$$\begin{array}{l} x-5=0 \\ x=5 \end{array}$$

$$\begin{array}{l} x+2=0 \\ x=-2 \end{array}$$