

4.3 Two-Way Tables & Venn Diagrams Part 2: Two-Way Tables & the General Addition Rule

When we found $P(\text{male and pierced ear})$ in part (b) of the example, we could have described this as either $P(A \text{ and } B)$ or $P(B \text{ and } A)$. Why? Because "A and B" describes the same event as "B and A". Likewise, $P(A \text{ or } B)$ is the same as $P(B \text{ or } A)$. Don't get so caught up in the notation that you lose sight of what's really happening!

Part (c) of the example reveals an important fact about finding the probability $P(A \text{ or } B)$: We can't use the addition rule for mutually exclusive events unless events A and B have no outcomes in common. In this case, there are 19 outcomes that are shared by events A and B - the students who are male and have a pierced ear. If we did add the probabilities of A and B, we'd get $90/178 + 103/178 = 193/178$.

This is clearly wrong because the probability is bigger than 1! As Figure 4.3 illustrates, outcomes common to both events are counted twice when we add the probabilities of these two events.

Gender	Pierced Ears?		Total
	Yes	No	
Male	19	71	90
Female	84	4	88
Total	103	75	178

We can fix the double-counting problem illustrated in the two-way table by subtracting the probability $P(\text{male and pierced ear})$ from the sum.

That is:

$$\begin{aligned} P(\text{male or pierced ear}) &= P(\text{male}) + P(\text{pierced ear}) - P(\text{male and pierced ear}) \\ &= 90/178 + 103/178 - 19/178 \\ &= 174/178 \end{aligned}$$

This result is known as the **general addition rule**.

If A and B are any two events resulting from some chance process, the **general addition rule** says that $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Example: A 2014 survey suggests that 71% of U.S. teenagers use Facebook, 33% use Twitter, and 15% do both. Suppose we select a U.S. teenager at random. What's the probability that the student uses Facebook or uses Twitter?

$$\begin{aligned}P(F \text{ or } T) &= P(F) + P(T) - P(F \& T) \\ &= 71 + 33 - 15 \\ &= 89\%\end{aligned}$$

As the example suggests, it is sometimes easier to designate events with letters that relate to the context, like F for "uses Facebook" and T for "uses Twitter".