11.4 Infinite Geometric Series

An infinite geometric series is sum of an infinite amount of terms given the first term and the common ratio.

Sum of an Infinite Geometric Series

$$S = \frac{a_1}{1-r} \text{ where } |r| < 1 \qquad \sum_{k=0}^{\infty} ar^k = \frac{a}{1-r}$$

a_1 r

If $|r| \geq 1$, the series has no sum.

Example 1

Find the sum of the infinite geometric series:

$-30 + 15 - \frac{15}{2} + \frac{15}{4} - \dots$

$\div -2 \quad \div -2 \quad \div -2$

$r = \frac{1}{-2}$

$$S = \frac{a_1}{1-r} = \frac{-30}{1 + \frac{1}{2}} = \frac{-30}{\frac{3}{2}} = \frac{-30}{1} \div \frac{3}{2} = \frac{-30}{1} \cdot \frac{2}{3} = \boxed{-20}$$

Example 2

Find the sum of the infinite geometric series given that $a_1 = 2$ and $r = \frac{6}{5} = 1\frac{1}{5}$

Can't have r be more than 1

DNE

Example 3

Find the sum of the infinite geometric series if it has one.

a.) $\sum_{i=1}^{\infty} \underbrace{\boxed{108}}_{a_1} \cdot \left(\overset{r}{\frac{1}{3}}\right)^{i-1}$

$S = \frac{a_1}{1-r} = \frac{108}{\frac{3}{3}1 - \frac{1}{3}}$

$= \frac{108}{2/3} = \frac{108}{1} \div \frac{2}{3}$

$= \frac{\cancel{108}^{54}}{1} \cdot \frac{3}{\cancel{2}} = \boxed{162}$

b.) $\sum_{i=1}^{\infty} \overset{a_1}{8}\left(\overset{r}{\frac{5}{4}}\right)^{i-1}$

$1\frac{1}{4} > 1$

$\boxed{\text{DNE}}$

Rewrite the infinite geometric series formula to solve for the common ratio.

$$(1-r)\cdot S = \frac{a_1}{1-r}\cdot(1-r)$$

$$\frac{S(1-r)}{S} = \frac{a_1}{S}$$

$$1 - r = \frac{a_1}{S} - 1$$
$$-1$$

$$\frac{-r}{-1} = \frac{\frac{a_1}{S}}{-1} \frac{-1}{-1}$$

$$\boxed{r = -\frac{a_1}{S} + 1}$$

Example 4

Find the common ratio given the sum $S = 6$ and $a_1 = 2$.

$$r = -\frac{a_1}{S} + 1 = -\frac{2}{6} + \cancel{1}\frac{6}{6} = \frac{4 \div 2}{6 \div 2} = \boxed{\frac{2}{3}}$$

11.5 Geometric Series.notebook