11.3 Geometric Sequences

multiply/divide

A geometric sequence is a sequence in which the ratio of any term to the previous term is constant. This constant ratio is called the common ratio and is denoted by *r*.

Decide whether each sequence is geometric.
If so, identify the common ratio and give the next 3 terms.

1. 1, 2, 6, 24, 120,...

×2 ×3 ×4 ×5

NO

2. 81, 27, 9, 3, 1,... $\frac{1}{3}, \frac{1}{9}, \frac{1}{27}$

÷3 ÷3 ÷3 ÷3

yes $r = \frac{1}{3}$

3. 4, -8, 16, -32, 64, -128, 256

×-2 ×-2 ×-2

yes $r = -2$

4. 3, 9, -27, -81, 243,...

×3 ×-3 ×3 ×-3

NO

*n*th Term of a Geometric Sequence

$$a_n = a_1 r^{n-1}$$

5. Find the eighth ($n=8$) term of the sequence defined by $a_1 = 3$ and $r = -2$.

$a_n = a_1 r^{n-1} = (3)(-2)^{8-1} = (3)(-2)^7 = (3)(-128)$

$= -384$

6. Find a_9 ($n=9$) of a geometric sequence if $a_1 = 144$ and $r = \frac{1}{2}$.

$a_n = a_1 r^{n-1} = (144)\left(\frac{1}{2}\right)^{9-1} = (144)\left(\frac{1}{2}\right)^8$

$= (144)\left(\frac{1}{256}\right)$

$= \frac{144 \div 2}{256 \div 2} = \frac{72 \div 2}{128 \div 2}$

$= \frac{36 \div 4}{64 \div 4} = \frac{9}{16}$

7. Find a_6 in the geometric sequence that includes $a_3 = 150$ and $a_5 = 3750$.

8. Write an explicit formula for the nth term of the geometric sequence $30, 10, 3\frac{1}{3}, 1\frac{1}{9}, \ldots$ $\div 3$ $r = \frac{1}{3}$

$$a_n = a_1 r^{n-1}$$

$$\boxed{a_n = (30)\left(\frac{1}{3}\right)^{n-1}}$$

11.3 Geometric Series

A geometric series is the indicated sum of the terms of a geometric sequence.

Sum of the First n Terms of a Geometric Series

$$S_n = t_1\left(\frac{1 - r^n}{1 - r}\right) \text{ where } r \neq 1$$

Example 1

Find S_7 of the geometric series $3 + 6 + 12 + 24 + \ldots$

$t_1 = 3$, $\times 2$, $\times 2$, $\times 2$, $r = 2$, $n = 7$

$$S_n = t_1\left(\frac{1 - r^n}{1 - r}\right)$$

$$S_7 = 3\left(\frac{1 - 2^7}{1 - 2}\right)$$

$$= 3\left(\frac{1 - 128}{-1}\right) = 3\left(\frac{-127}{-1}\right)$$

$$= 3(127) = \boxed{381}$$

Example 2

Find S_{12} of the geometric series 1 + (-4) + 16 + (-64) +...

t_1 $r=-4$ $n=12$

$$S_n = t_1\left(\frac{1-r^n}{1-r}\right)$$

$$S_{12} = 1\left(\frac{1-(-4)^{12}}{1+4}\right) = \frac{1-16777216}{5}$$

$$= \frac{-16777215}{5}$$

$$= \boxed{-3355443}$$

Example 3

Given the series 4 + 2 + 1 +..., find S_{10}.

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