11.2 ARITHMETIC SEQUENCES

adding / subtracting

In an *arithmetic sequence*, the difference between consecutive terms in *constant*. The *constant difference* is called the *common difference* and is denoted by *d*.

Examples: Decide whether each sequence is arithmetic. If so, identify the common difference.

1. -7, -3, 1, 5, 9,... yes $d=4$
   +4 +4 +4 +4
2. 2, 3, 5, 8, 12, 17,... no
   +1 +2 +3 +4 +5
3. -14, -8, -2, 0, 2, 8, 14,... no
   +6 +6 +2 +2 +6 +6
4. 19, 13, 7, 1, -5,... yes $d=-6$
   -6 -6 -6 -6

The *n*th term of an arithmetic sequence with first term a_1 and common difference *d* is given by:

$$a_n = a_1 + (n-1)d$$

5. Write an explicit formula for the *n*th term of the arithmetic sequence if $d = -12$ and $a_1 = 80$.

$$a_n = a_1 + (n-1)d$$

$$a_n = 80 + (n-1)(-12)$$

$$a_n = \boxed{80} + -12n \boxed{+12}$$

$$\boxed{a_n = -12n + 92}$$

6. Write an explicit formula for the nth term of the arithmetic sequence 50, 44, 38, 32,...

$-6 \quad -6 \quad -6 \qquad d=-6$

$$a_n = a_1 + (n-1)d$$

$$a_n = 50 + (n-1)(-6)$$

$$a_n = \boxed{50} + -6n \boxed{+6}$$

$$\boxed{a_n = -6n + 56}$$

7. Write an explicit formula for the nth term of the arithmetic sequence shown below. Then find the 20th term.

32, 47, 62, 77,... $d=15$

$+15 \quad +15 \quad +15$

$$a_n = a_1 + (n-1)d$$

$$a_n = 32 + (n-1)(15)$$

$$a_n = \boxed{32} + 15n \boxed{-15}$$

$$\boxed{a_n = 15n + 17}$$

$n=20 \qquad a_{20} = 15(20)+17 = 300+17$

$$\boxed{a_{20} = 317}$$

8. Write an explicit formula for the *n*th term of the arithmetic sequence shown below. Then find the 25th term.

4, 1, -2, -5, -8,...

11.2 *Arithmetic Series*

An *arithmetic series* is the indicated sum of the terms of an arithmetic sequence.

For example, consider the sequence 1, 2, 3,... The sum of the first six terms is denoted by S_6.

$S_6 = 1 + 2 + 3 + 4 + 5 + 6 = 21$

Example 1: Find S_8 for the arithmetic series $\underbrace{2 + 5}_{+3} \underbrace{+ 8}_{+3} + \ldots$

$$2 + 5 + 8 + 11 + 14 + 17 + 20 + 23$$

+3 +3 +3 +3 +3 +3 +3

$$\boxed{100}$$

Sum of the First *n* Terms of an Arithmetic Series

1st term, last term

$$S_n = n\left(\frac{t_1 + t_n}{2}\right)$$

Example 2: Given 3 + 15 + 27 + ..., find S_{10}.

t_1 +12 +12 $d=12$ $n=10$

$a_n = a_1 + (n-1)d$

$a_{10} = 3+(10-1)(12)$

$a_{10} = 3+(9)(12)$

$a_{10} = 3+108$

$a_{10} = 111$

$S_{10} = 10\left(\frac{3+111}{2}\right)$

$= 10\left(\frac{114}{2}\right) = 10(57)$

$\boxed{S_{10} = 570}$

Example 3: Find S_{11} given the arithmetic series 16 + 12 + 8 +...

Derive a new formula for S_n to find the number of terms n.

$$S_n = n\left(\frac{t_1 + t_n}{2}\right)$$

Example 4: Find the sum of the arithmetic series
-4 + (-13) + (-22) + ... + (-76).

Example 5: Find the sum of the arithmetic series
$14 + 17 + 20 + \ldots + 65$.

Example 6: Evaluate $\sum_{k=1}^{12} (6 - 2k)$.

12st term

$n=12$

1st term

$S_n = n\left(\frac{t_1 + t_n}{2}\right)$

$t_1 = 6 - 2(1) = 6 - 2 = 4$

$t_{12} = 6 - 2(12) = 6 - 24 = -18$

$S_n = 12\left(\frac{4 + -18}{2}\right)$

$= 12\left(\frac{-14}{2}\right) = 12(-7)$

$\boxed{S_{12} = -84}$

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