11.1 SEQUENCES & SERIES

A sequence is an ordered list of numbers called terms.

2, 4, 6, 8, 10,...   3, -9, 27, -81, 243,...   5, 2, -1, -4, -7

ellipsis - *indicates that a sequence is an* infinite sequence, *which continues without end*

→ *domain is the set of all natural numbers* {1, 2, 3,...,*n*,...}

a finite sequence *has a last term*

→ *domain is the set of all natural numbers* {1, 2, 3,...,*n*}

A formula that defines the nth term of a sequence is called an explicit formula.

EXAMPLES

1. Write the first six terms of the sequence defined by the explicit formula $t_n = -2n + 3$.

$t_1 = -2 \cdot 1 + 3 = -2 + 3 = 1$

$t_2 = -2 \cdot 2 + 3 = -4 + 3 = -1$

$t_3 = -2 \cdot 3 + 3 = -6 + 3 = -3$

$t_4 = -2 \cdot 4 + 3 = -8 + 3 = -5$

$t_5 = -2 \cdot 5 + 3 = -10 + 3 = -7$

$t_6 = -2 \cdot 6 + 3 = -12 + 3 = -9$

EXAMPLES

2. Write the first six terms of the sequence defined by the explicit formula $t_n = 2n^2 - 1$.

$t_1 = 2 \cdot 1^2 - 1 = 2 \cdot 1 - 1 = 2 - 1 = 1$

$t_2 = 2 \cdot 2^2 - 1 = 2 \cdot 4 - 1 = 8 - 1 = 7$

$t_3 = 2 \cdot 3^2 - 1 = 2 \cdot 9 - 1 = 18 - 1 = 17$

$t_4 = 2 \cdot 4^2 - 1 = 2 \cdot 16 - 1 = 32 - 1 = 31$

$t_5 = 2 \cdot 5^2 - 1 = 2 \cdot 25 - 1 = 50 - 1 = 49$

$t_6 = 2 \cdot 6^2 - 1 = 2 \cdot 36 - 1 = 72 - 1 = 71$

EXAMPLES

3. Write the first six terms of the sequence defined by the explicit formula $f(n) = \frac{n+2}{2n}$.

$f(1) = \frac{1+2}{2 \cdot 1} = \frac{3}{2}$

$f(2) = \frac{2+2}{2 \cdot 2} = \frac{4}{4} = 1$

$f(3) = \frac{3+2}{2 \cdot 3} = \frac{5}{6}$

$f(4) = \frac{4+2}{2 \cdot 4} = \frac{6}{8} = \frac{3}{4}$

$f(5) = \frac{5+2}{2 \cdot 5} = \frac{7}{10}$

$f(6) = \frac{6+2}{2 \cdot 6} = \frac{8}{12} = \frac{2}{3}$

A sequence can also be defined by a recursive formula. With a recursive formula, one or more previous terms are used to generate the next term.

For Example 1, the recursive formula is $t_1 = 1$ and $t_n = t_{n-1} - 2$, where $n \geq 2$.

$$t_n = t_{n-1} - 2 \quad \text{and} \quad t_1 = 1$$

$$t_2 = t_1 - 2 = 1 - 2 = -1$$

$$t_3 = t_2 - 2 = -1 - 2 = -3$$

$$t_4 = t_3 - 2 = -3 - 2 = -5$$

4. Write the first six terms of the sequence defined by the recursive formula $t_1 = 1$ and $t_n = 3t_{n-1} - 1$, where $n \geq 2$.

1st term

$$t_1 = 1$$

$$t_2 = 3 \cdot t_1 - 1 = 3 \cdot 1 - 1 = 3 - 1 = 2$$

$$t_3 = 3 \cdot t_2 - 1 = 3 \cdot 2 - 1 = 6 - 1 = 5$$

$$t_4 = 3 \cdot t_3 - 1 = 3 \cdot 5 - 1 = 15 - 1 = 14$$

$$t_5 = 3 \cdot t_4 - 1 = 3 \cdot 14 - 1 = 42 - 1 = 41$$

$$t_6 = 3 \cdot t_5 - 1 = 3 \cdot 41 - 1 = 123 - 1 = 122$$

SERIES

A series is an expression that indicates the sum of terms of a sequence.

EXAMPLE: For the sequence 2, 4, 6, 8, 10, the resulting expression is the series 2 + 4 + 6 + 8 + 10 = 30.

Summation notation, which uses the Greek letter sigma, Σ, is a way to express a series in abbreviated form.

2, 4, 6, 8, 10

$$\sum_{n=1}^{5} 2n$$

5 "end"

n = 1 "Start"

Values of n from 1 to 5 are called the index.

Explicit formula

"the sum of 2n for the values of n from 1 to 5"

5. Write the terms of the series. Then evaluate. $\sum_{k=1}^{4} 5k$

$t_1 = 5 \cdot 1 = 5$

$t_2 = 5 \cdot 2 = 10$

$t_3 = 5 \cdot 3 = 15$

$t_4 = 5 \cdot 4 = 20$

$5+10+15+20=\boxed{50}$

Write the terms of the series. Then evaluate.

6. $\sum_{j=1}^{3} 7$

$t_1 = 7$

$t_2 = 7$

$t_3 = 7$

$7+7+7=\boxed{21}$

7. $\sum_{n=1}^{5} (3n+1)$

$t_1 = 3\cdot1+1 = 3+1 = 4$

$t_2 = 3\cdot2+1 = 6+1 = 7$

$t_3 = 3\cdot3+1 = 9+1 = 10$

$t_4 = 3\cdot4+1 = 12+1 = 13$

$t_5 = 3\cdot5+1 = 15+1 = 16$

$4+7+10+13+16 = \boxed{53}$

8. $\sum_{m=1}^{4} (2m^2+m)$

$t_1 = 2\cdot1^2+1 = 2\cdot1+1 = 2+1 = 3$

$t_2 = 2\cdot2^2+2 = 2\cdot4+2 = 8+2 = 10$

$t_3 = 2\cdot3^2+3 = 2\cdot9+3 = 18+3 = 21$

$t_4 = 2\cdot4^2+4 = 2\cdot16+4 = 32+4 = 36$

$3+10+21+36 = \boxed{70}$

Evaluate the sum when n = 5.

9. $\sum_{k=1}^{\cancel{n}\,5} \frac{2}{k+1}$

$t_1 = \frac{2}{1+1} = \frac{2}{2} = 1$

$t_2 = \frac{2}{2+1} = \frac{2}{3}$

$t_3 = \frac{2}{3+1} = \frac{2}{4} = \frac{1}{2}$

$t_4 = \frac{2}{4+1} = \frac{2}{5}$

$t_5 = \frac{2}{5+1} = \frac{2}{6} = \frac{1}{3}$

$\frac{1\cdot30}{1\cdot30} + \frac{2\cdot10}{3\cdot10} + \frac{1\cdot15}{2\cdot15} + \frac{2\cdot6}{5\cdot6} + \frac{1\cdot10}{3\cdot10}$

$\frac{30}{30} + \frac{20}{30} + \frac{15}{30} + \frac{12}{30} + \frac{10}{30}$

$\boxed{\frac{87}{30}}$

10. $\sum_{r=2}^{\cancel{n}\,5} \frac{r}{r+2}$

$t_2 = \frac{2}{2+2} = \frac{2}{4} = \frac{1}{2}$

$t_3 = \frac{3}{3+2} = \frac{3}{5}$

$t_4 = \frac{4}{4+2} = \frac{4}{6} = \frac{2}{3}$

$t_5 = \frac{5}{5+2} = \frac{5}{7}$

$\frac{1\cdot105}{2\cdot105} + \frac{3\cdot42}{5\cdot42} + \frac{2\cdot70}{3\cdot70} + \frac{5\cdot30}{7\cdot30}$

$\frac{105}{210} + \frac{126}{210} + \frac{140}{210} + \frac{150}{210}$

$\boxed{\frac{521}{210}}$