4.7 The Multiplication Counting Principle & Permutations

Finding the probability of an event often involves counting the number of possible outcomes of some chance process. In this lesson, we will show you two techniques for determining the number of ways that a multi-step process can happen when the order of the steps matters.

The Agricola Restaurant offers a three-course dinner menu. Customers who order from this menu must choose one appetizer, one main dish, and one dessert. The options are: (appetizer) soup, green salad, Caesar salad, (main dish) pork chop, steak, chicken, salmon, (dessert) cake, and tart.

How many different meals can be ordered from this menu?

We see that there are 3 choices of appetizers, 4 choices of main dishes, and 2 choices of desserts. So there are:

$$3 \times 4 \times 2 = 24$$

different meals that can be ordered from the three-course dinner menu. This is an example of the **multiplication counting principle**.

For a process involving multiple (k) steps, suppose that there are n_1 ways to do Step 1, n_2 ways to do Step 2,..., and n_k ways to do Step k. The total number of different ways to complete the process is

$$n_1 \times n_2 \times \ldots \times n_k$$

This result is called the **multiplication counting principle**.

Example: The standard license plate for California passenger cars has one digit, followed by three letters, and then three more digits. The first digit cannot be a 0. The first and third letters cannot be I, O, or Q. How many possible license plates are there?

$\underset{\text{cannot be } 0}{\underline{9}} \cdot \underset{\text{cannot be I, O, Q}}{\underline{23}} \cdot \underline{26} \cdot \underset{\text{cannot be I, O, Q}}{\underline{23}} \cdot \underline{10} \cdot \underline{10} \cdot \underline{10}$

$\boxed{123{,}786{,}000}$

The multiplication counting principle can also help us determine how many ways there are to arrange a group of people, animals, or things. For example, suppose you have 5 framed photographs of different family members that you want to arrange in a line on top of your dresser. In how many ways can you do this? Let's count the options moving from left to right across the dresser. There are 5 options for the first photo, 4 options for the next photo, and so on. By the multiplication counting principle, there are
5 x 4 x 3 x 2 x 1 = 120 different photo arrangements.

We call arrangements like this, where the order matters, **permutations**.
A **permutation** is a distinct arrangement of some group of individuals.

Expressions like 5 x 4 x 3 x 2 x 1 occur often enough in counting problems that mathematicians invented a special name and notation for them. We write:
5 x 4 x 3 x 2 x 1 = 5! (read as "5 factorial").

Example: The manager of a youth baseball team has picked 9 players to start an upcoming playoff game. How many different ways are there for the manager to arrange these 9 players to make up the team's batting order?

$$9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$\boxed{362{,}880}$$

So far, we have shown how to count the number of distinct arrangements of *all* the individuals in a group of people, animals, or things. Sometimes, we want to determine how many ways there are to select and arrange only *some* of the individuals in a group.

Mr. Wilcox likes to get the students in his statistics class involved in the action. But he doesn't want to play favorites. Each day, Mr. Wilcox puts the names of all 28 of his students in a hat and mixes them up. He then draws out 3 names, one at a time. The student whose name is picked first gets to operate the display calculator for the day. The second student selected is in charge of reading the answers to the even-numbered homework problems. The third student picked writes class notes on the interactive whiteboard. In how many different ways can Mr. Wilcox fill these three jobs?

By the multiplication counting principle, there are 28 x 27 x 26 = 19,656 ways for Mr. Wilcox to fill the three different jobs.

Example: A youth baseball team has 15 players. How many different ways are there for the team's manager to select and arrange 9 of these players to make up the team's batting order?

$$\underline{15} \cdot \underline{14} \cdot \underline{13} \cdot \underline{12} \cdot \underline{11} \cdot \underline{10} \cdot \underline{9} \cdot \underline{8} \cdot \underline{7}$$

$$\boxed{1{,}816{,}214{,}400}$$