

Example 7

Simplify:  $\frac{5x}{3x-12} \div \frac{x^2-2x}{x^2-6x+8}$

$$\begin{array}{c} \xrightarrow{\text{red}} 5x \\ \hline \xrightarrow{\text{blue}} 3x-12 \end{array} \cdot \frac{\overset{\text{orange}}{1}x^2-6x+8}{x^2-2x} \quad \leftarrow \begin{array}{l} \text{orange } 5-6 \mid 8 \\ -2+(-4) \mid -2 \cdot -4 \end{array}$$

$$\frac{\overset{\text{red}}{5}x}{\underset{\text{blue}}{3}(x-4)} \cdot \frac{\overset{\text{orange}}{(x-2)}(x-4)}{\underset{\text{green}}{x}(x-2)}$$

$$\frac{\cancel{5x} \cancel{(x-2)} \cancel{(x-4)}}{\cancel{3x} \cancel{(x-4)} \cancel{(x-2)}} = \boxed{\frac{5}{3}}$$

Example 8

Simplify:  $\frac{3}{4x-8} \div \frac{x^2+3x}{x^2+x-6}$

$$\rightarrow \frac{3}{4x-8} \cdot \frac{x^2+x-6}{x^2+3x}$$

Sum:  $-2+3$  | prod:  $-6$   
 $-2 \cdot 3$

$$\frac{3}{4(x-2)} \cdot \frac{(x-2)(x+3)}{x(x+3)}$$

$$\frac{3 \cancel{(x-2)} \cancel{(x+3)}}{4x \cancel{(x-2)} \cancel{(x+3)}} = \boxed{\frac{3}{4x}}$$

Example 9

Simplify:  $\frac{6x^2 + 7x - 3}{6x^2} \div \frac{(2x^2 + 3x)}{1}$

$\frac{6x^2 + 7x - 3}{6x^2} \cdot \frac{1}{2x^2 + 3x}$

$\frac{(2x+3)(3x-1)}{6x^2} \cdot \frac{1}{x(2x+3)}$

$\frac{(2x+3)(3x-1)}{6x^3(2x+3)} = \boxed{\frac{3x-1}{6x^3}}$

Handwritten notes:
 

- AC method for  $6x^2 + 7x - 3$ :  $\begin{matrix} 5 & 7 & | & p & -18 \\ 9 & +2 & | & q & -2 \end{matrix}$
- Factorization:  $\frac{3}{2} \cdot -1 = -\frac{1}{3}$
- Canceling factors:  $\frac{6x^2}{6x^2}$  and  $\frac{2x+3}{2x+3}$  are crossed out.
- Arrows indicate the cancellation of  $6x^2$  and  $2x+3$ .

Example 10

Simplify:  $\frac{8x^2 + 10x - 3}{4x^2} \div \frac{(4x^2 - x)}{1}$

S 10 | p. -24  
12 + -2 | 12 · -2

$$\frac{12}{8}$$

$$\frac{72}{8}$$

$$\frac{3}{2}$$

$$\frac{-1}{4}$$

$$\frac{8x^2 + 10x - 3}{4x^2}$$

$$\frac{(2x+3)(4x-1)}{4x^2}$$

$$\frac{1}{4x^2 - x}$$

$$\frac{1}{x(4x-1)}$$

$$\frac{(2x+3)\cancel{(4x-1)}}{4x^3\cancel{(4x-1)}} = \boxed{\frac{2x+3}{4x^3}}$$

Example 11

Simplify:  $\frac{x}{x+5} \cdot (3x-5) \div \frac{9x^2-25}{x+5}$

$$\begin{array}{l} \rightarrow \frac{x(3x-5)}{x+5} \cdot \frac{x+5}{9x^2-25} \\ \rightarrow \frac{x(3x-5)}{x+5} \cdot \frac{x+5}{(3x-5)(3x+5)} \end{array}$$

← difference of squares

$$\frac{x \cancel{(3x-5)} \cancel{(x+5)}}{\cancel{(x+5)} \cancel{(3x-5)} (3x+5)} = \boxed{\frac{x}{3x+5}}$$

Example 12

Simplify:  $\frac{x}{x-2} \cdot (2x+3) \div \frac{4x^2-9}{x-2}$

$$\frac{x(2x+3)}{x-2} \cdot \frac{x-2}{4x^2-9}$$

← difference of Squares

$$\frac{x(2x+3)}{x-2} \cdot \frac{x-2}{(2x-3)(2x+3)}$$

$$\frac{\cancel{x(2x+3)}(x-2)}{(x-2)\cancel{(2x-3)}\cancel{(2x+3)}} = \boxed{\frac{x}{2x-3}}$$