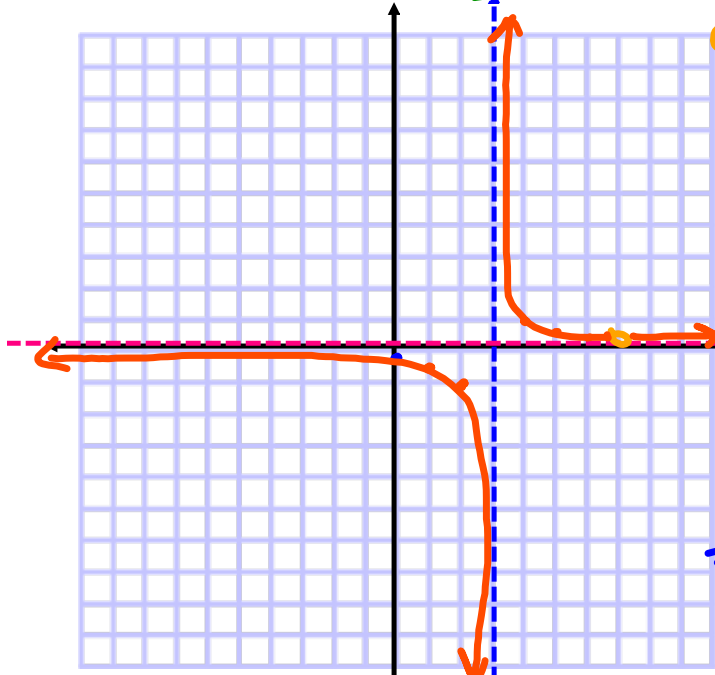


Example:  $f(x) = \frac{x^{\textcircled{1}} - 7}{x^{\textcircled{2}} - 10x + 21} = \frac{x-7}{(x-7)(x-3)} = \frac{1}{x-3}$

*S-10 | p-21*  
*-7+3 | -7-3*



Common Holes:  $x-7=0 \Rightarrow x=7, y=\frac{1}{7-3}=\frac{1}{4}$

leftover in VA:  $x-3=0 \Rightarrow *x=3*$

original HA:  $y=0$   
"bottom heavy"

\*no x-int

y-int:  $y = \frac{0-7}{(0-7)(0-3)} = \frac{-7}{(-7)(-3)} = \frac{-7}{21}$

$y = -\frac{1}{3}$

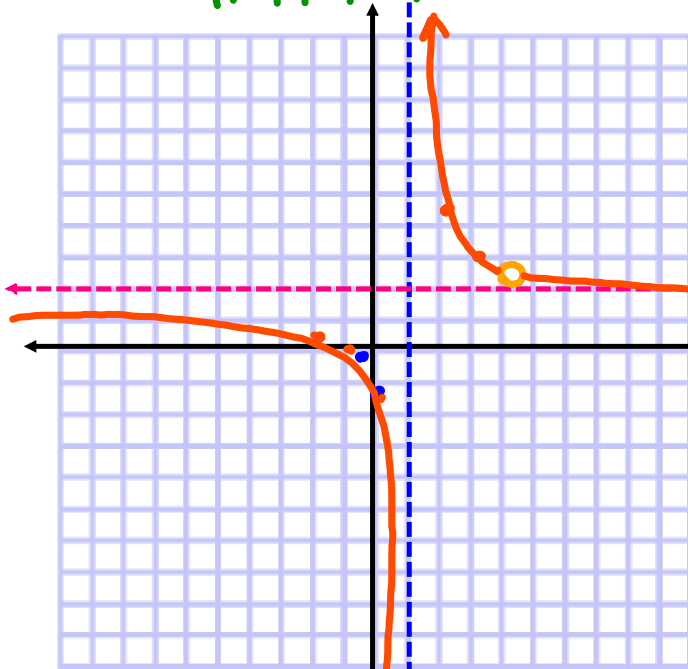
before 3  $y = \frac{1}{x-3}$

after 3

x	
0	$\frac{1}{0-3} = -\frac{1}{3}$
1	$\frac{1}{1-3} = -\frac{1}{2}$
2	$\frac{1}{2-3} = \frac{1}{-1} = -1$

x	
4	$\frac{1}{4-3} = \frac{1}{1} = 1$
5	$\frac{1}{5-3} = \frac{1}{2}$
6	$\frac{1}{6-3} = \frac{1}{3}$

Example:  $y = \frac{2x^2 - 7x - 4}{x^2 - 5x + 4} = \frac{(x-4)(2x+1)}{(x-1)(x-4)} = \frac{2x+1}{x-1}$



Common Holes:  $x-4=0$   
 $x=4$

leftover in bottom VA:  $x-1=0$   
 $*x=1*$

degrees HA:  $y = \frac{2}{1} = 2$   
"Same"

x-int:  $0 = \frac{2x+1}{x-1}$

$0 = 2x+1$

$-\frac{1}{2} = \frac{2x}{2}$

$x = -\frac{1}{2}$

y-int:  $y = \frac{2 \cdot 0 + 1}{0 - 1} = \frac{0+1}{-1}$

$y = \frac{1}{-1} = -1$

$y = -1$

before 1

after 1

x	y
-2	$\frac{2 \cdot -2 + 1}{-2 - 1} = \frac{-3}{-3} = 1$
-1	$\frac{2 \cdot -1 + 1}{-1 - 1} = \frac{-1}{-2} = \frac{1}{2}$
0	-1

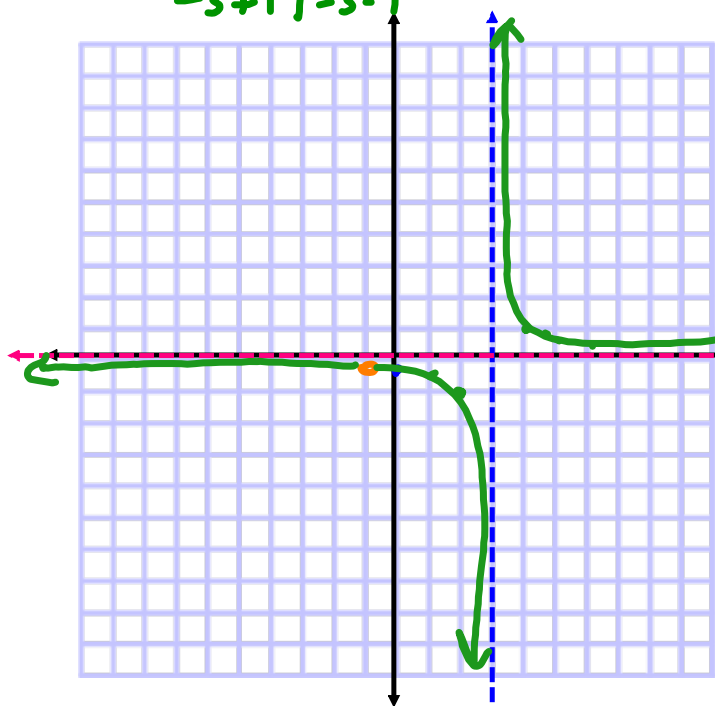
y-int

x	y
2	$\frac{2 \cdot 2 + 1}{2 - 1} = \frac{5}{1} = 5$
3	$\frac{2 \cdot 3 + 1}{3 - 1} = \frac{7}{2} = 3\frac{1}{2}$
4	$\frac{2 \cdot 4 + 1}{4 - 1} = \frac{9}{3} = 3$

hole

Example:  $y = \frac{x^0 + 1}{x^2 - 2x - 3} = \frac{x+1}{(x-3)(x+1)} = \frac{1}{x-3}$

*S: -2 / p: -3*  
*-3+1 / -3-1*



Common Holes:  $x+1=0$   $y = \frac{1}{-1} = -1$   
 $x = -1$   $y = \frac{1}{-4}$

leftover in VA:  $x-3=0$   $*x=3*$   
 bottom

degree HA:  $y=0$   
 "bottom heavy"

\*no x-int

y-int:  $y = \frac{1}{0-3} = -\frac{1}{3}$

before 3

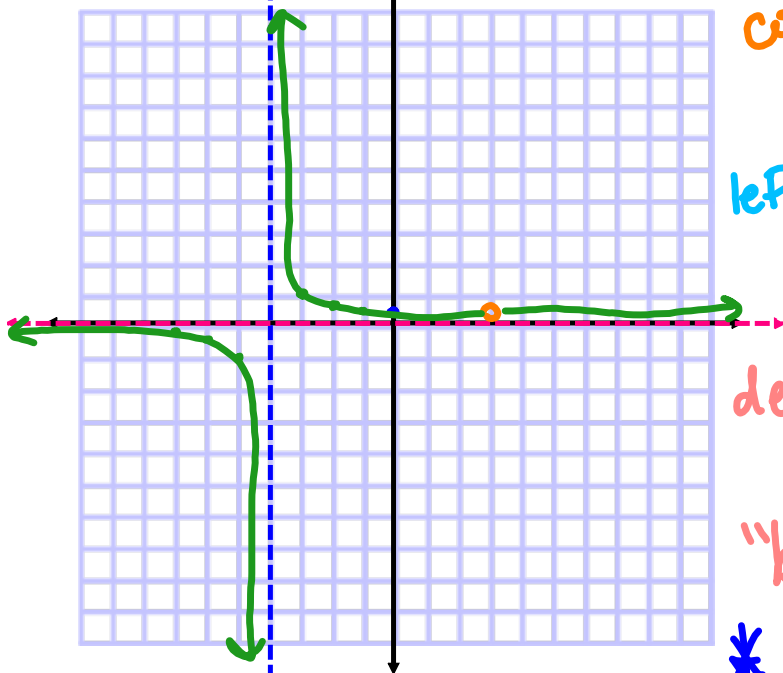
after 3

x	
0	$\frac{1}{0-3} = -\frac{1}{3}$
1	$\frac{1}{1-3} = -\frac{1}{2}$
2	$\frac{1}{2-3} = \frac{1}{-1} = -1$

x	
4	$\frac{1}{4-3} = \frac{1}{1} = 1$
5	$\frac{1}{5-3} = \frac{1}{2}$
6	$\frac{1}{6-3} = \frac{1}{3}$

Example:  $y = \frac{x^{\textcircled{1}} - 3}{x^{\textcircled{2}} + x - 12} = \frac{x - 3}{(x - 3)(x + 4)} = \frac{1}{x + 4}$

S. 1 | p. -12  
4 + -3 | 4 · -3



Common

Holes:  $x - 3 = 0 \Rightarrow y = \frac{1}{3+4}$   
 $x = 3 \Rightarrow y = \frac{1}{7}$

leftover

VA:  $x + 4 = 0$   
 $* x = -4 *$

degree

HA:  $y = 0$   
"bottom heavy"

\* no x-int.

y-int:  $y = \frac{1}{0+4} = \frac{1}{4}$   
 $\frac{y}{(x=0)}$

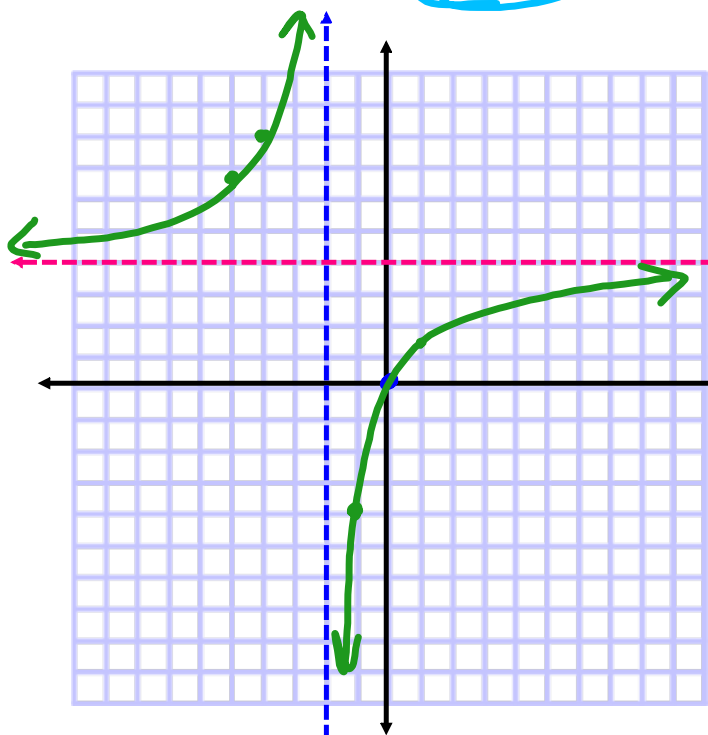
before -4

x	y
-7	$\frac{1}{-7+4} = \frac{1}{-3}$
-6	$\frac{1}{-6+4} = \frac{1}{-2}$
-5	$\frac{1}{-5+4} = \frac{1}{-1} = -1$

after -4

x	y
-3	$\frac{1}{-3+4} = \frac{1}{1} = 1$
-2	$\frac{1}{-2+4} = \frac{1}{2}$
-1	$\frac{1}{-1+4} = \frac{1}{3}$

Example:  $y = \frac{4x}{x+2}$



Common  
Holes: none

leftover  
VA:  $x+2=0$   
 ~~$x = -2$~~

degree  
HA:  $y = \frac{4}{1} = 4$   
Same

x-int:  $(x+2) \cdot 0 = \frac{4x}{x+2}$   ~~$(x+2)$~~   
(y=0)

$$\frac{0}{4} = \frac{4x}{4}$$

$$x=0$$

y-int:  $y = \frac{4 \cdot 0}{0+2} = \frac{0}{2} = 0$   
(x=0)

before -2

after -2

x	
-5	$\frac{4 \cdot -5}{-5+2} = \frac{-20}{-3} = \frac{20}{3} = 6\frac{2}{3}$
-4	$\frac{4 \cdot -4}{-4+2} = \frac{-16}{-2} = 8$
-3	$\frac{4 \cdot -3}{-3+2} = \frac{-12}{-1} = 12$

x	
-1	$\frac{4 \cdot -1}{-1+2} = \frac{-4}{1} = -4$
0	$\frac{4 \cdot 0}{0+2} = \frac{0}{2} = 0$
1	$\frac{4 \cdot 1}{1+2} = \frac{4}{3} = 1\frac{1}{3}$

## Attachments

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Graph Rational Functions.doc