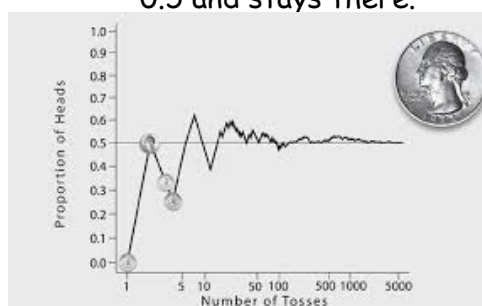


### 4.1 Randomness, Probability, and Simulation: Idea of Probability

Chance is all around us. You & your friend play rock-paper-scissors to determine who gets the last slice of pizza. A coin toss decides which team gets to receive the ball first in a football game. People young & old play games of chance involving cards, dice, or spinners. The traits that children inherit (gender, hair, eye color, blood type, handedness, dimples, whether or not they can roll their tongues) are determined by the chance involved in which genes their parents pass along.

The mathematics of chance behavior is called *probability*. These figures show some results of tossing a coin & the proportion of heads from those tosses. The proportion of tosses that land heads varies in the first 10 tosses. As we make more & more tosses, however, the proportion of heads gets closer to 0.5 and stays there.



When we watch coin tosses or the results of random sampling and random assignment closely, a remarkable fact emerges: *Chance behavior is unpredictable in the short run but has a regular and predictable pattern in the long run.* This is the basis for the idea of **probability**.

The probability of any outcome of a chance process is a number between 0 & 1 that describes the proportion of times the outcome would occur in a very large number of repetitions.

Outcomes that never occur have probability 0. An outcome that happens on every repetition has probability 1. An outcome that happens half the time in a very long series of trials has probability of 0.5.

The fact that the proportion of heads in many tosses eventually closes in on 0.5 is guaranteed by the **law of large numbers**.

The **law of large numbers** says that if we observe more and more repetitions of any chance process, the proportion of times that a specific outcome occurs approaches its probability.

Life-insurance companies, casinos, and other who make important decisions based on probability rely on the long-run predictability of chance behavior.

**Example:** According to *The Book of Odds*, the probability that a randomly selected US adult usually eats breakfast is 0.61.

a.) Explain what probability  $0.61$  means in this setting.

61% of U.S. adults eat  
breakfast.

b.) Does this probability say that if 100 US adults are chosen at random, exactly 61 of them usually eat breakfast? Explain.

No because the people were  
Chosen at random so there  
is a possibility for people who  
do not eat breakfast.