

**Try these:** Find the vertical asymptotes and holes, if any.  
 Drag each function to the shaded box below to check your answer!

*not in common*  
*in the denominator*

*Common*

$\frac{4x-3}{x(x-6)}$   
 $x-6=0$   
 $x=6$   
 $x=0$

$\frac{57x+12}{3x+4}$

$\frac{3x(x+4)}{(x+3)(x+4)}$   
 $x+4=0$   
 $x=-4$   
 $x+3=0$   
 $x=-3$

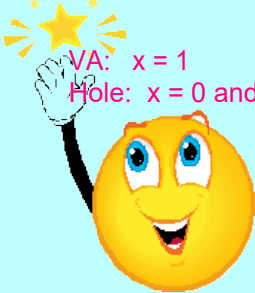
$\frac{x(1+x)}{x(x-1)(x+1)}$   
 $x=0$   
 $x+1=0$   
 $x=-1$   
 $x-1=0$   
 $x=1$

$f(x) = \frac{4x-3}{x^2-6x}$        $f(x) = \frac{3x^2+12x}{x^2+7x+12}$        $f(x) = \frac{x+x^2}{x^3-x}$

VA:  $x=0$  and  $x=6$   
 Hole: none

VA:  $x=-3$   
 Hole:  $x=-4$

VA:  $x=1$   
 Hole:  $x=0$  and  $x=-1$



B. Asymptotes and Holes of Rational Functions (continued)

3. Horizontal Asymptotes (HA)

*highest exponents*

To find the horizontal asymptote (HA) of a rational function, you MUST compare the *degree* of the numerator to the degree of the denominator.

*\* (Only look at the term with the largest exponent in both the numerator and denominator.) \**

3 Cases:

Let  $n$  = degree of numerator,  
 and let  $d$  = degree of denominator.

*"bottom heavy"*

*degree is same*

*"top heavy"*

1. If  $n < d$ , then  $y = 0$  is the HA.

2. If  $n = d$ , then  $y = a/b$  is the HA  
 ("a" and "b" are the coefficients of the leading terms in the numerator and denominator)

3. If  $n > d$ , then there is NO HA.

$f(x) = \frac{2x-3}{3x+4}$   
 $y = \frac{2}{3}$

Examples: Find the horizontal asymptote, if any, of each function.

1.  $y = \frac{x^1}{x^2 - 2x - 3}$  **HA:  $y = 0$**   
 "bottom heavy"

Click  
 here

2.  $f(x) = \frac{2x^1 + 2}{3x^1 - 2}$  **HA:  $y = \frac{2}{3}$**   
 degrees are same

Click  
 here

3.  $f(x) = \frac{x^2 - 4}{x^1 + 5}$  **NO HA**  
 "top heavy"

Click  
 here

Try these:

Erase below problem to reveal answer !!!

Find the horizontal asymptote, if any, of each function.

1.  $f(x) = \frac{-3}{x^2 - 3x}$  **HA:  $y = 0$**   
 "bottom heavy"

2.  $y = \frac{1x^1 + 1}{1 - 1x^1}$  **HA:  $y = \frac{1}{-1} = -1$**   
 Same

3.  $f(x) = \frac{x^2}{2x^2}$  **HA:  $y = \frac{1}{2}$**   
 Same

Putting It ALL together! Well.....almost all!  
 Find the domain, all asymptotes and holes.



*"bottom heavy"*

$$1. f(x) = \frac{3x-1}{x^2-9}$$

$$f(x) = \frac{3x-1}{\underline{(x-3)}\underline{(x+3)}}$$

$\neq$  D:  $x-3=0$   
 $x \neq 3$

$x+3=0$   
 $x \neq -3$

*• don't have in common*  
*• denom. same*  
*- Common:*

VA:  $x-3=0$   
 $x=3$

$x+3=0$   
 $x=-3$

Hole: NO holes

*look at original* HA:  $y=0$

Click to check your answers.

Find the domain, all asymptotes and holes.

$$2. f(x) = \frac{x^2 - 6x + 9}{x^2 - 5x + 6}$$

D:

Click to check your answers.

VA:

Hole:

HA:

### C. Intercepts



1. Y-Intercept: substitute  $x = 0$  into the equation.
  - a. If "y" exists, this is the value of the y-intercept.
  - b. If "y" is undefined, there is NO y-intercept.  
*denominator is zero*

2. X-Intercepts (if any): set the numerator equal to 0 and solve for the x values.

NOW!!!!

*Sum -2 / prod. -15  
-5+3 / -5·3*

Find the domain, all asymptotes, holes and intercepts.

$$3. \quad y = \frac{x^3 - 2x^2 - 15x}{x(x+3)}$$

$$y = \frac{x(x^2 - 2x - 15)}{x(x+3)}$$

Click to check your answers.

$$y = \frac{x(x+3)(x-5)}{x(x+3)}$$



D:  $x \neq 0$

$x + 3 = 0$   
 $x \neq -3$

VA: NONE

Hole:  $x = 0$

$x + 3 = 0$   
 $x = -3$

HA: *top heavy* none

x-intercept:  $x - 5 = 0$   
 $x = 5$

y-intercept: none

*left over in the top*  
*Set x=0*

$\frac{x(x+3)(x-5)}{x(x+3)}$

$y = \frac{0(0+3)(0-5)}{0(0+3)}$   
 $y = \frac{0(3)(-5)}{0(3)}$

*undefined*  
 $y = \frac{0}{\neq}$

## Attachments

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Practice 8-2 Rational Functions.doc