9.2-9.3 Part 1 Rational Functions

Rational functions are used in science and engineering to model complex equations in areas such as

1) fields and forces in physics,
2) electronic circuitry,
3) aerodynamics,
4) medicine concentrations,
5) optics to improve image resolution, and
6) acoustics and sound.

Rational Functions and Their Graphs

- A rational function is the quotient of two polynomial functions.
- The equation of a rational function looks like

$$f(x) = \frac{p(x)}{q(x)}$$

where p(x) and q(x) are polynomials
AND q(x) is **NOT** zero.

EXAMPLE:

$$f(x) = \frac{2}{6+x}$$

p(x) is the numerator 2 AND
q(x) is the denominator (6 + x)

Which of the functions below are rational functions?

Drag each function to the shaded box below to check your answer!

$$y = \frac{x+2}{2x^2 + 3x - 2}$$

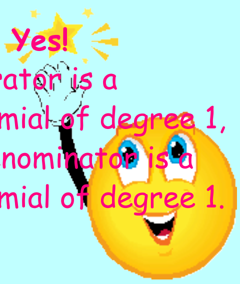
$$y = \frac{x^2 + 2}{|x|}$$

$$y = \frac{3x}{x-4}$$

Yes!
Numerator is a polynomial of degree 1, and denominator is a polynomial of degree 2.

No!
Denominator is NOT a polynomial.

Yes!
Numerator is a polynomial of degree 1, and denominator is a polynomial of degree 1.



A. Domain of Rational Functions

D = all reals, EXCEPT..... any number that makes the denominator equal 0.

To find the exceptions (also called excluded values) for the domain set the denominator equal to 0 and solve for the variable.

Examples: Find the domain of each function.

1. $f(x) = \dfrac{x}{2x-7}$

$2x - 7 = 0$

$+7 \quad +7$

$\dfrac{2x}{2} = \dfrac{7}{2}$

$\boxed{x \neq \dfrac{7}{2}}$

Examples continued:
Find the domain of each rational function.

2. $f(x) = \dfrac{x}{\boxed{(x-7)}\boxed{(x+3)}}$

$x-7=0$
$+7 \quad +7$
$\boxed{x \neq 7}$

$x+3=0$
$-3 \quad -3$
$\boxed{x \neq -3}$

3. $y = \dfrac{x^2-2}{x^2-9x-36} = \dfrac{x^2-2}{\boxed{(x-12)}\boxed{(x+3)}}$

Sum -9	prod. -36
$-12+3$	$-12 \cdot 3$

$\dfrac{-12}{1} \quad \dfrac{3}{1}$

$(x-12)(x+3)$

$x-12=0$
$+12 \quad +12$
$\boxed{x \neq 12}$

$x+3=0$
$-3 \quad -3$
$\boxed{x \neq -3}$

Pull

Pull

Examples continued:
Find the domain of each rational function

4. $y = \frac{12-2x}{x^2-4}$

$x^2 - 4 = 0$
$+4 \quad +4$
$\sqrt{x^2} = \sqrt{4}$
$\boxed{x \neq \pm 2}$

5. $f(x) = \frac{x^2-5x+4}{2x^2-7x-4} = \frac{x^2-5x+4}{\boxed{(x-4)}\boxed{(2x+1)}}$

Sum $\cancel{-7}$	prod. -8
$-8+1$	$-8 \cdot 1$

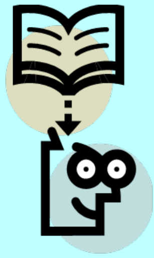
$\frac{-4}{1} = \cancel{\frac{-8}{2}} \quad \frac{1}{2}$

$(x-4)(2x+1)$

$x - 4 = 0$
$+4 \quad +4$
$\boxed{x \neq 4}$

$2x + 1 = 0$
$-1 \quad -1$
$\frac{2x}{2} = \frac{-1}{2}$
$\boxed{x \neq -\frac{1}{2}}$

B. Asymptotes and Holes of Rational Functions



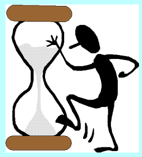
1. Vertical Asymptotes
   May occur at excluded values of the domain.

To find the vertical asymptotes (VA) of a rational function:

1. Factor numerator and denominator, if possible.
2. Identify factors of the denominator that are **NOT** factors of the numerator.
3. Set each identified factor equal to 0 and solve for the variable.

☆ These x-values are the vertical asymptotes!!! ☆

Examples: Find the vertical asymptotes, if any.



1. $f(x)=\dfrac{2x}{x^2-1}=\dfrac{2x}{(x-1)(x+1)}$ VA: x = 1 and x = -1

$x-1=0$, $+1\ \ +1$, $x=1$

$x+1=0$, $-1\ \ -1$, $x=-1$

2. $f(x)=\dfrac{3x}{x^2-3x+2}=\dfrac{3x}{(x-1)(x-2)}$ VA: x = 2 and x = 1

S -3 | P 2

$-2+-1$ | $-2\cdot-1$

$\frac{-2}{1}$ $\frac{-1}{1}$

$x-1=0$, $+1\ \ +1$, $x=1$

$x-2=0$, $+2\ \ +2$, $x=2$

3. $f(x)=\dfrac{2x+4}{x^2-3x-10}=\dfrac{2(x+2)}{(x-5)(x+2)}$

S -3 | P -10

$-5+2$ | $-5\cdot 2$

$\frac{2}{1}$ $\frac{-5}{1}$

$x-5=0$, $+5\ \ +5$, $x=5$

B. Asymptotes and Holes of Rational Functions (continued)

2. Holes

If a factor of the denominator IS a factor of the numerator, then a hole in the graph occurs.

Examples: Find the values of x for any holes in the graph of each function.

1. $f(x)=\frac{2x}{x^2-4x} = \frac{2x}{x(x-4)}$ $\boxed{x=0}$ hole

2. $f(x)=\frac{3x}{x^2-3x+2} = \frac{3x}{(x-2)(x-1)}$ NO holes

S -3 | P. 2
-2+-1 | -2·-1

nothing in common

3. $f(x)=\frac{2x-1}{2x^2+5x-3} = \frac{2x-1}{(x+3)(2x-1)}$

S. 5 | P. -6
6+-1 | 6·-1

$\frac{3}{1} = \cancel{\frac{6}{2}}$ $\frac{-1}{2}$

$2x-1=0$
$+1 \quad +1$
$\frac{2x}{2} = \frac{1}{2}$

hole → $\boxed{x=\frac{1}{2}}$

Attachments

Practice 8-2 Rational Functions.doc