

Combined Variation

Combined variation is any combination of direct, inverse, and/or joint variation.

Ex: b varies jointly as c and e and inversely as d

$$b = \frac{kce}{d}$$

multiply divide
Constant of Variation

Example 5

Write a general equation for each.

a) y varies directly with b and inversely with c

$$y = \frac{kb}{c}$$

b) z varies jointly as x and the cube root of z

$$z = Kx\sqrt[3]{z}$$

c) h varies inversely as g and jointly as e and f

$$h = \frac{kef}{g}$$

Example 6

$$y = \frac{kxz}{w}$$

The variable y varies jointly as x and z , and inversely as w . If $y = 72$ when $x = 6$, $z = 3$, and $w = \frac{1}{2}$:

- a) Find the constant of variation. (find k)
 b) Write the appropriate combined variation equation.
 c) Find y when $w = 18$, $x = \frac{1}{4}$, and $z = 12$.

$$\textcircled{a} \quad y = \frac{kxz}{w}$$

$$72 = \frac{k \cdot 6 \cdot 3}{\frac{1}{2}}$$

$$\frac{1}{2} \cdot 72 = \frac{k \cdot 18}{\cancel{\frac{1}{2}}} \cdot \cancel{\frac{1}{2}}$$

$$\frac{36}{18} = \frac{k \cdot 18}{18}$$

$$\boxed{k = 2}$$

$$\textcircled{b} \quad y = \frac{2xz}{w}$$

$$\textcircled{c} \quad y = \frac{2xz}{w}$$

$$y = \frac{2 \cdot \frac{1}{4} \cdot 12}{18}$$

$$y = \frac{2 \cdot 3}{18}$$

$$y = \frac{6 \div 6}{18 \div 6}$$

$$\boxed{y = \frac{1}{3}}$$

Example 7

The variable a varies directly as b , and inversely as the square root of c . If $a = 10$ when $b = 6$ & $c = 9$:

- a) Find the constant of variation. (find k)
 b) Write the appropriate combined variation equation.
 c) Find b when $a = 20$ and $c = 4$.

$$\textcircled{a} \quad a = \frac{kb}{\sqrt{c}}$$

$$10 = \frac{k \cdot 6}{\sqrt{9}}$$

$$10 = \frac{k \cdot 6}{3}$$

$$\frac{10}{2} = \frac{k \cdot 2}{2}$$

$$\boxed{k = 5}$$

$$\textcircled{b} \quad a = \frac{5b}{\sqrt{c}}$$

$$\textcircled{c} \quad a = \frac{5b}{\sqrt{c}}$$

$$20 = \frac{5b}{\sqrt{4}}$$

$$2 \cdot 20 = \frac{5b}{2} \cdot 2$$

$$\frac{40}{5} = \frac{5b}{5}$$

$$\boxed{b = 8}$$

Example 8 $m = \frac{kjp^3}{n}$

The variable m varies jointly as j and the cube of p , and inversely as n . If $m = 30$ when $j = 5$, $p = 2$, & $n = 4$:

- a) Find the constant of variation. (find k)
 b) Write the appropriate combined variation equation.
 c) Find m when $j = 4$, $p = 3$, and $n = 2$.

$$\textcircled{a} \quad m = \frac{kjp^3}{n}$$

$$30 = \frac{k \cdot 5 \cdot 2^3}{4}$$

$$30 = \frac{k \cdot 5 \cdot 8}{4}$$

$$30 = \frac{k \cdot 40}{4}$$

$$\frac{30}{10} = \frac{k \cdot 10}{10}$$

$$\boxed{k = 3}$$

$$\textcircled{b} \quad m = \frac{3jp^3}{n}$$

$$\textcircled{c} \quad m = \frac{3jp^3}{n}$$

$$m = \frac{3 \cdot 4 \cdot 3^3}{2}$$

$$m = \frac{3 \cdot 4 \cdot 27}{2}$$

$$m = \frac{12 \cdot 27}{2}$$

$$m = 6 \cdot 27$$

$$\boxed{m = 162}$$