

## 9.1 Direct, Inverse, Joint, and Combined Variation

## RECALL from Algebra 1: Direct Variation

If  $y$  varies directly as  $x$ , then  $y = kx$ .

## Example 1

The variable  $y$  varies directly as  $x$ ,  
and  $y = 6$  when  $x = 2$ .

↑ constant  
of  
variation

a) Find the constant of variation. (find  $k$ )

b) Write the appropriate inverse variation equation.

c) Find  $y$  when  $x$  is  $\frac{1}{2}$ ,  $1$ , and  $2$ .

↑ plug  $k$  in

a)  $y = kx$   
 $\frac{6}{2} = \frac{k \cdot 2}{2}$   
 $3 = k$

b)  $y = 3x$

c)  $y = 3 \cdot \frac{1}{2}$   
 $y = \frac{3}{2}$  or  $\frac{1}{2}$  or 1.5  


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 $y = 3 \cdot 1$   
 $y = 3$   


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 $y = 3 \cdot 2$   
 $y = 6$

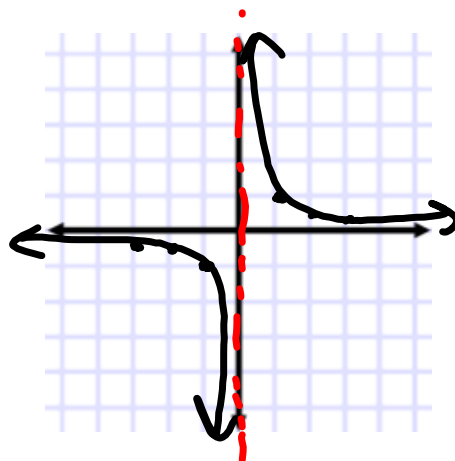
Inverse Variation

If  $y$  varies inversely as  $x$ , then  $y = \frac{k}{x}$ .

## Example 2

Graph  $y = \frac{1}{x}$ .

$x$		$y$
-3	$-\frac{1}{3}$	-0.3
-2	$-\frac{1}{2}$	-0.5
-1	$-\frac{1}{1}$	-1
<del>0</del>	<del>0</del>	<del>undefined</del>
1	$\frac{1}{1}$	1
2	$\frac{1}{2}$	0.5
3	$\frac{1}{3}$	0.3



## Example 3

The variable  $y$  varies inversely as  $x$ ,  
and  $y = 6$  when  $x = 3$ .

$$y = \frac{k}{x}$$

- a) Find the constant of variation. (find  $k$ )  
 b) Write the appropriate inverse variation equation.  
 c) Find  $y$  when  $x$  is  $\frac{1}{2}$ ,  $1$ , and  $2$ .

a)  $y = \frac{k}{x}$   
 $3 \cdot 6 = \frac{k}{3} \cdot 3$

$$18 = k$$

b)  $y = \frac{18}{x}$

c)  $y = \frac{18}{\frac{1}{2}} = \frac{18 \cdot 2}{1} = \frac{36}{1}$

$$y = 36$$

$$y = \frac{18}{1} = 18$$

$$y = 18$$

$$y = \frac{18}{2} = 9$$

$$y = 9$$

Joint Variation

If  $y$  varies jointly as  $x$  and  $z$ , then  $y = kxz$ .

## Example 4

The variable  $y$  varies jointly as  $x$  and  $z$ ,  
and  $y = 16$  when  $x = 4$  and  $z = \frac{1}{2}$ .

- Find the constant of variation. (find  $k$ )
- Write the appropriate joint variation equation.
- Find  $y$  when  $x = 2$  and  $z = \frac{1}{4}$ .

$$\textcircled{a} \quad y = kxz$$

$$16 = k \cdot 4 \cdot \frac{1}{2}$$

$$\frac{4 \cdot \frac{1}{2} = \frac{4}{2} = 2$$

$$\frac{16}{2} = \frac{k \cdot 2}{2}$$

$$\boxed{8 = k}$$

$$\textcircled{b} \quad \boxed{y = 8xz}$$

$$\textcircled{c} \quad y = 8xz$$

$$y = 8 \cdot 2 \cdot \frac{1}{4}$$

$$y = 16 \cdot \frac{1}{4}$$

$$y = \frac{16}{4} = \frac{4}{1} = 4$$

$$\boxed{y = 4}$$