Example 7

Solve each equation. Check for extraneous solutions.

a) $16 \ln x = 30$

b) $\ln (x + 1) = 5$

$e^{\ln(x+1)} = e^5$

$x + 1 = e^5$

$-1 \quad -1$

$\boxed{x = e^5 - 1}$

$\ln(e^5 - 1 + 1) = 5 \Rightarrow \ln e^5 = 5$

$5 = 5$ ✓

c) $\ln (x - 4) = -8$

$e^{\ln(x-4)} = e^{-8}$

$x - 4 = e^{-8}$

$+4 \quad +4$

$\boxed{x = e^{-8} + 4}$

$\ln(e^{-8} + 4 - 4) = -8$

$\ln e^{-8} = -8$

$-8 = -8$ ✓

d) $-18 \ln x = 9$

$-18 \quad -18$

$\ln x = -\frac{1}{2}$

$e^{\ln x} = e^{-\frac{1}{2}}$

$\boxed{x = e^{-\frac{1}{2}}}$

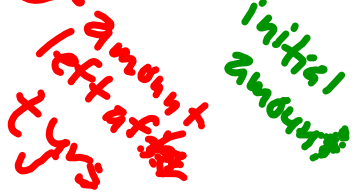
$-18 \ln e^{-\frac{1}{2}} \stackrel{?}{=} 9$

$-18 \cdot -\frac{1}{2} \stackrel{?}{=} 9$

$9 = 9$ ✓

Most of the carbon found in the Earth's atmosphere is the isotope carbon-12, but a small amount is the radioactive isotope carbon-14. Plants absorb carbon dioxide from the atmosphere, and animals obtain carbon from the plants they consume. When a plant or animal dies, the amount of carbon-14 it contains decays in such a way that exactly half of its initial amount is present after 5730 years. The function below models the decay of carbon-14, where N_0 is the initial amount of carbon-14 and N(t) is the amount present t years after the plant or animal dies.

$$N(t) = N_0 e^{-0.00012t}$$



Example 8 $N(t) = N_0 e^{-0.00012t}$

Suppose that archeologists find scrolls and claim that they are 2000 years old. Tests indicate that the scrolls contain .78% of their original carbon-14. Could the scrolls be 2000 years old?

multiply

N_0 $t = ?$

$$\frac{0.78N_0}{N_0} = \frac{N_0 e^{-0.00012t}}{N_0}$$

$$0.78 = e^{-0.00012t}$$

$$\ln 0.78 = \ln e^{-0.00012t}$$

$$\frac{\ln 0.78}{-0.00012} = \frac{-0.00012t}{-0.00012}$$

$$t = \frac{\ln 0.78}{-0.00012} \approx 2070.511327$$

≈ 2071 yrs

Yes, the scrolls could be 2000 yrs old.

$t \approx 2071$ yrs

Example 9 $N(t) = N_0 e^{-0.00012t}$

Suppose that a jar containing grain is found at an archeological dig and that archeologists claim that it is 6500 years old. Tests indicate that the grain contains 62% of its original carbon-14. Could the grain be 6500 years old?

$$\frac{0.62 N_0}{N_0} = \frac{N_0 e^{-0.00012t}}{N_0}$$

$$0.62 = e^{-0.00012t}$$

$$\ln 0.62 = \ln e^{-0.00012t}$$

$$\frac{\ln 0.62}{-0.00012} = \frac{-0.00012t}{-0.00012}$$

No, the dirt could not be 6500 yrs old.

$$t = \frac{\ln 0.62}{-0.00012} \approx 3983.631675$$

$t \approx 3984$ yrs