

8.4-8.6 The Natural Logarithm

Recall that we learned about e^x , the *natural exponential function*.

The *natural logarithm* is a logarithm with a *base of e*.

$$\begin{array}{c} \log_e x \\ \downarrow \\ \text{written without a base} \\ \downarrow \\ \ln x \end{array}$$

Example 1

Evaluate each using a calculator.

a) $\ln 8$ b) $\ln 0.04$ c) $\ln (-5)$ d) $\ln 100$
 2.079 -3.219 undefined 4.605

Recall that logarithms are *inverses* of exponential functions.

So the inverse of e^x is $\ln x$.

This means $\ln e^x = x$ and $e^{\ln x} = x$.

Example 2

Simplify each expression.

a) $e^{\ln 9}$ b) $\ln e^{x/2}$ c) $e^{\ln 20}$ d) $3 \ln e^4$
 9 $\frac{x}{2}$ 20 3 · 4
 12

The properties of logarithms apply to the natural logarithm also.

$$\ln ab = \ln a + \ln b$$

$$\ln \frac{a}{b} = \ln a - \ln b$$

$$2 \ln a = \ln a^2$$

Example 3

Expand each expression.

a) $\ln x^2 y$

$$\ln x^2 + \ln y$$

$$2 \ln x + \ln y$$

b) $\ln \frac{a}{4b^2}$

$$\ln a - \ln 4b^2$$

$$\ln a - \ln 4 + \ln b^2$$

$$\ln a - \ln 4 + 2 \ln b$$

$$\ln a - \ln 4 + 2 \ln b$$

Example 4

$$\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

Condense each expression.

a) $\ln 20 + 2 \ln \frac{1}{2} + \ln x$

$$\ln 20 + \ln \left(\frac{1}{2}\right)^2 + \ln x$$

$$\ln 20 + \ln \frac{1}{4} + \ln x = \ln \left(20 \cdot \frac{1}{4}\right) x$$

b) $4 \ln 2 - 3 \ln m + \frac{1}{2} \ln n$

$$\ln 2^4 - \ln m^3 + \ln n^{\frac{1}{2}}$$

$$\ln 16 - \ln m^3 + \ln \sqrt{n}$$

$$\ln \frac{16}{m^3 \sqrt{n}}$$

$$= \boxed{\ln 5x}$$

Example 5Solve each equation using the natural logarithm function

a) $14^x = 20$

$$\ln 14^x = \ln 20$$

$$\frac{x \ln 14}{\ln 14} = \frac{\ln 20}{\ln 14}$$

$$x = 1.135$$

b) $18^{x/2} = 5$

$$\ln 18^{x/2} = \ln 5$$

$$2 \cdot \frac{x}{2} \ln 18 = \ln 5$$

$$\frac{x \ln 18}{\ln 18} = \frac{2 \ln 5}{\ln 18}$$

$$x = 1.114$$

Example 6

Write an equivalent exponential or logarithmic equation.

a) $e^x = 16$

$$\ln e^x = \ln 16$$

$$x = \ln 16$$

b) $e^x = 44$

$$\ln e^x = \ln 44$$

$$x = \ln 44$$

c) $\ln 8 \approx 2.079$

$$e^{\ln 8} \approx e^{2.079}$$

$$8 \approx e^{2.079}$$

d) $\ln \frac{1}{4} \approx -1.386$

$$e^{\ln \frac{1}{4}} \approx e^{-1.386}$$

$$\frac{1}{4} \approx e^{-1.386}$$

Example 7

Solve each equation. Check for extraneous solutions.

a) $16 \ln x = 30$

$$\frac{16}{16} \ln x = \frac{30}{16}$$

$$\ln x = \frac{15}{8}$$

$$e^{\ln x} = e^{\frac{15}{8}}$$

$$x = e^{\frac{15}{8}}$$

$$x = 6.521$$

b) $\ln(x + 1) = 5$

c) $\ln(x - 4) = -8$

d) $-18 \ln x = 9$