8.1 - 8.2 Part 3 Exponential Growth and Decay Word Problems

Exponential Growth Equation: * $y = a(1 + r)^t$ *

a is the initial amount

r is the percent increase, written as a decimal

1 + r is the growth factor

EXAMPLE 1:

In 1995, there were $\boxed{275}$ cell phone subscribers in Aiken. The number of subscribers increased by $\boxed{75\%}$ per year after 1995. Write an exponential equation that models the number of cell phone subscribers after t years. How many were there in $\boxed{2004?}$

a

$r = 0.75$

$t = 9$

$$y = a(1+r)^t$$
$$y = 275(1+0.75)^t$$
$$y = 275(1.75)^t$$
$$y = 275(1.75)^9$$
$$y = 42332.61843$$
$$\boxed{y \approx 42333}$$

EXAMPLE 2:

$$y = a(1+r)^t$$

In the exponential equation $y = \boxed{35}(\boxed{1.27})^x$, identify:

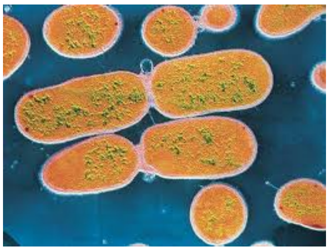
a) the initial amount, 35

b) the growth factor, 1.27

c) the percent increase.

$$1 + r = 1.27$$
$$-1 \qquad -1$$
$$r = 0.27 \Rightarrow \boxed{27\%}$$

EXAMPLE 3: $100\% + 100\% = 200\% = 2.00$



Bacteria can multiply at an alarming rate when each bacteria splits into two new cells, thus doubling. If a scientist starts with three bacteria which can double every hour, how many bacteria will she have by the end of the day?

$a = 3$

$$y = a(1+r)^t$$

$$y = 3(1+2)^{24}$$

$$y = 3(3)^{24}$$

$$\boxed{y = 8.47 \times 10^{11}}$$

EXAMPLE 4: $y = a(1+r)^t$

In 1970, the population of a city was about 278,000. Since then, the city population has grown at an average annual rate of 1.8%. $r = 0.018$

a) Write an exponential equation that models the population of this city t years after 1970.

$$y = 278000(1+0.018)^t = 278000(1.018)^t$$

b) About how many people lived in the city in 1990? $t = 20$

$$y = 278000(1.018)^{20} \approx 397192$$

c) What is the population of this city today? $t = 51$

$$y = 278000(1.018)^{51} = 690528$$

Exponential Decay Equation: $y = a(1 - r)^t$

a is the initial amount

r is the percent decrease, written as a decimal

1 - r is the decay factor

EXAMPLE 5:

Jolene purchases a new car for $\boxed{\$22,499.}$ (a) The value of the car decreases by $\boxed{11\%}$ each year. Write the exponential equation that models the car's value after t years. Then find its value after $\boxed{3 \text{ years.}}$ $t=3$

$r=0.11$

$$y=a(1-r)^t$$
$$y=22499(1-0.11)^t$$
$$y=22499(0.89)^t$$
$$y=22499(0.89)^3$$
$$\boxed{y \approx \$15861.10}$$

EXAMPLE 6: $y=a(1-r)^t$

In the exponential equation $y = \boxed{200}(\boxed{0.71})^x$, identify:

a) the initial amount, 200

b) the decay factor, 0.71

c) the percent decrease.

$$0.71 = 1 - r$$
$$-1 \qquad -1$$
$$\frac{-0.29}{-1} = \frac{-r}{-1}$$
$$r = 0.29 = \boxed{29\%}$$

EXAMPLE 7: $y = a(1-r)^t$

An adult takes $\boxed{400}$ mg of ibuprofen. Each hour, the amount of ibuprofen in the person's system decreases by about $\boxed{29\%.}$ $r = 0.29$

a) Write an exponential equation that models the amount of ibuprofen left in this adult's system after t hours.

$$y = 400(1-0.29)^t = 400(0.71)^t$$

b) How much ibuprofen is left after $\boxed{6 \text{ hours?}}$ $t=6$

$$y = 400(0.71)^6 = 51.24 \text{ mg}$$

EXAMPLE 8:

You drink a beverage with $\boxed{120 \text{ mg}}$ of caffeine. Each hour, the caffeine in your system decreases by about $\boxed{12\%.}$ $\rightarrow r = 0.12$ Estimate the amount of caffeine in your system after $\boxed{7 \text{ hours.}}$ $t = 7$

$$y = a(1-r)^t$$

$$y = 120(1-0.12)^7$$

$$y = 120(0.88)^7$$

$$y = 49.04 \text{ mg}$$