

8.6 Part 2 Solving Exponential Equations

Solving Equations Without a Calculator

Sometimes we can solve exponential equations without a calculator simply by changing the bases.

Example 1: Solve $7^{4x} = 49^{4x-1}$. $7^2 = 49$

$$7^{4x} = (7^2)^{4x-1}$$

$$7^{4x} = 7^{8x-2}$$

$$4x = 8x - 2$$

$$\frac{-4x}{-4} = \frac{-2}{-4} \quad \frac{-2}{-4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

$$x = \frac{1}{2}$$

Example 2: Solve $9^{2x} = 27^{x-1}$.

$$3^2 = 9 \quad 3^3 = 27$$

$$(3^2)^{2x} = (3^3)^{x-1}$$

$$3^{4x} = 3^{3x-3}$$

$$4x = 3x - 3$$

$$x = -3$$

Example 3: Solve $4^{2x-5} = 16^{x+6}$.

$$4^2 = 16$$

$$4^{2x-5} = (4^2)^{x+6}$$

$$4^{2x-5} = 4^{2x+12}$$

$$2x-5 = 2x+12$$

$$-2x \quad -2x$$

$$-5 \neq 12$$

NO SOLUTION

Example 4: Solve $100^{7x+1} = 1000^{3x-2}$.

$$10^2 = 100$$

$$10^3 = 1000$$

$$(10^2)^{7x+1} = (10^3)^{3x-2}$$

$$10^{14x+2} = 10^{9x-6}$$

$$14x+2 = 9x-6$$

$$-9x \quad -9x$$

$$5x+2 = -6$$

$$5x = -8$$

$$\frac{5x}{5} = \frac{-8}{5}$$

$$x = \frac{-8}{5}$$

Example 5: Solve $81^{3-x} = \left(\frac{1}{3}\right)^{5x-6}$.

$$3^4 = 81 \quad 3^{-1} = \frac{1}{3}$$

$$\left(3^4\right)^{3-x} = \left(3^{-1}\right)^{5x-6}$$

$$3^{12-4x} = 3^{-5x+6}$$

$$12-4x = -5x+6$$

$$\quad +5x \quad +5x$$

$$12+x = 6$$

$$\quad -12 \quad -12$$

$$x = -6$$

Example 6: Solve $4^x = \left(\frac{1}{2}\right)^{x-3}$.

$$2^2 = 4 \quad 2^{-1} = \frac{1}{2}$$

$$\left(2^2\right)^x = \left(2^{-1}\right)^{x-3}$$

$$2^{2x} = 2^{-x+3}$$

$$2x = -x+3$$

$$\quad +x \quad +x$$

$$\frac{3x}{3} = \frac{3}{3}$$

$$x = 1$$

Example 7: Solve $3^x + 2 = 29$.

$$-2 \quad -2$$

$$3^x = 27$$

$$3^x = 3^3$$

$$x = 3$$

Example 8: Solve $16^{x-5} - 5 = 59$.

$$+5 \quad +5$$

$$16^{x-5} = 64$$

$$(4^2)^{x-5} = 4^3$$

$$4^{2x-10} = 4^3$$

$$2x - 10 = 3$$

$$+10 \quad +10$$

$$\frac{2x}{2} = \frac{13}{2}$$

$$x = \frac{13}{2}$$