8.6 Part 1 Solving Logarithmic Equations

One-to-One Property of Logarithms

If $\log_b x = \log_b y$, then $x = y$.

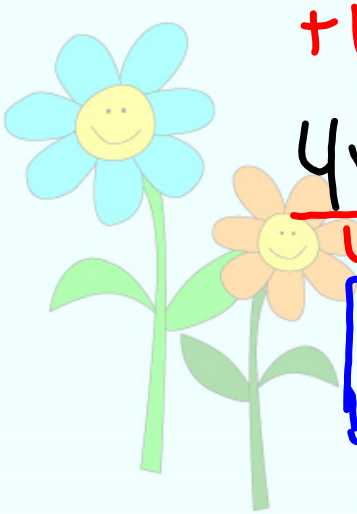
When solving these equations,
you must check for extraneous solutions!

In other words, make sure the solution
allows us to take the log of a positive value.

Example 1

Solve $\log_3(5x - 1) = \log_3(x + 7)$.

$$5x - 1 = x + 7$$
$$-1x \quad -x$$
$$4x - 1 = 7$$
$$+1 \quad +1$$
$$\frac{4x}{4} = \frac{8}{4}$$
$$x = 2$$



Check:

$\log_3(5x-1) = \log_3(x+7)$

$\log_3(5 \cdot 2 - 1) = \log_3(2+7)$

$\log_3(10-1) = \log_3(9)$

$\log_3(9) \overset{\checkmark}{=} \log_3(9)$

Example 2

Solve $\log_4(x + 3) = \log_4(8x + 17)$.

$x+3 = 8x+17$

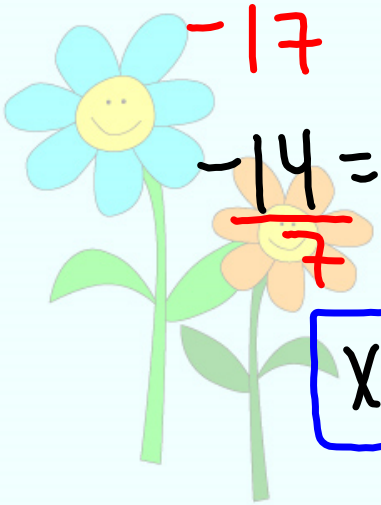
$-x \quad -x$

$3 = 7x+17$

$-17 \quad -17$

$\frac{-14}{7} = \frac{7x}{7}$

$\boxed{x=-2}$



Check:

$\log_4(x+3)=\log_4(8x+17)$

$\log_4(-2+3)=\log_4(8(-2)+17)$

$\log_4(1)=\log_4(-16+17)$

$\log_4(1)=\log_4(1)$ ✓

Example 3

Solve $\log_5 x = \log_5(2x + 7)$.

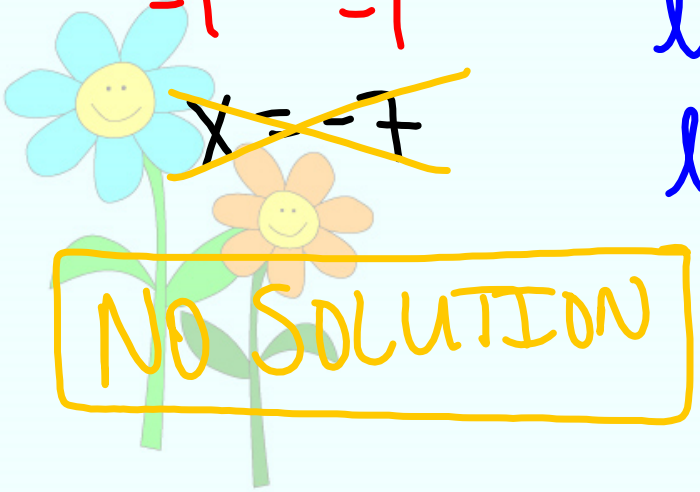
$$x = 2x + 7$$

$$-2x \quad -2x$$

$$-1x = 7$$

$$\frac{-1x}{-1} = \frac{7}{-1}$$

$$x = -7$$



NO SOLUTION

Check:

$$\log_5 x = \log_5(2x+7)$$

$$\log_5(-7) = \log_5(2 \cdot -7 + 7)$$

$$\log_5(-7) = \log_5(-14+7)$$

$$\log_5(-7) \neq \log_5(-7)$$

Cannot have negatives in the log

Example 4

Solve $\log_a(x^2 + 7x - 5) = \log_a(6x + 1)$.

$x^2+7x-5=6x+1$

$-6x \quad -6x$

$x^2+x-5=1$

$-1 \quad -1$

$1x^2+1x-6=0$

Sum 1	prod. -6
3+-2	3·-2

$3 \quad -2$

$1 \quad 1$

$(1x+3)(1x-2)=0$

$x+3=0$ $\quad x-2=0$

$-3 \quad -3 \quad +2 \quad +2$

$x=-3$ $\quad x=2$

Check:

$\log_2((-3)^2+7\cdot-3-5)=\log_2(6\cdot-3+1)$

$\log_2(9-21-5)=\log_2(-18+1)$

$\log_2(-17)=\log_2(-17)$

$\log_2(2^2+7\cdot2-5)=\log_2(6\cdot2+1)$

$\log_2(4+14-5)=\log_2(12+1)$

$\log_2(13)=\log_2(13)$ ✓

Example 5

Solve $\log_9 2x^2 = \log_9(5x + 3)$.

$$2x^2 = 5x + 3$$
$$-5x \quad -5x$$
$$2x^2 - 5x = 3$$
$$-3 \quad -3$$
$$2x^2 - 5x - 3 = 0$$

Sum -5	prod. -6
$-6+1$	$-6 \cdot 1$

$\frac{-3}{1}$ $\quad \frac{+1}{2}$ $\quad \frac{1}{2}$

$$(1x-3)(2x+1) = 0$$

$x - 3 = 0$
$+3 \quad +3$
$x = 3$

$2x + 1 = 0$
$-1 \quad -1$
$\frac{2x}{2} = \frac{-1}{2}$
$x = -\frac{1}{2}$

Check:

$\log_9(2 \cdot (3)^2) = \log_9(5 \cdot 3 + 3)$

$\log_9(2 \cdot 9) = \log_9(15 + 3)$

$\log_9(18) = \log_9(18)$ ✓

$\log_9\left(2 \cdot \left(-\frac{1}{2}\right)^2\right) = \log_9\left(5 \cdot -\frac{1}{2} + \frac{3}{1}\right)$

$\log_9\left(2 \cdot \frac{1}{4}\right) = \log_9\left(\frac{-5}{2} + \frac{6}{2}\right)$

$\log_9\left(\frac{1}{2}\right) = \log_9\left(\frac{1}{2}\right)$ ✓