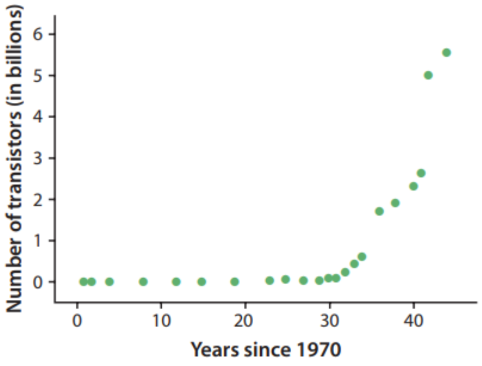
2.8 Fitting Models to Curved Relationships Part 2: Exponential Models

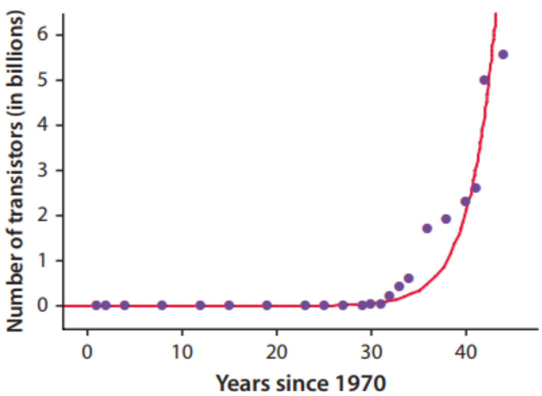
An **exponential model** is a model in the form $\hat{y} = a(b)^x$ where $b > 0$.
If $b > 1$, the graph will show exponential growth.
If $0 < b < 1$, the graph will show exponential decay.



Gordon Moore, one of the founders of Intel Corporation, predicted in 1965 that the number of transistors on an integrated circuit chip would double every 18 months. This is Moore's law, one way to measure the revolution of computing. This scatterplot shows the number of transistors for Intel microprocessors & the number of years since 1970.

If Moore's prediction is correct, then an exponential model should be a good model for the data.

The same scatterplot here includes the exponential model: $\hat{y} = 1277\,(1.431)^x$.
The exponential model fits the form of this association fairly well.



In 2014, there were 5,560,000,000 transistors on the Intel 18-core Xeon Haswell-E5 microprocessor. The exponential model suggests that it would have $\hat{y} = 1277(1.431)^{44} = 9{,}001{,}912{,}406$ transistors.

The residual for this value is:
5,560,000,000 - 9,001,912,406 = -3,441,912,406 transistors.

In other words, it has about 3.4 billion fewer transistors than expected, based on the exponential model.

Example: A random sample of 14 golfers was selected from the 147 players on the Ladies Professional Golf Association (LPGA) tour in a recent year. The total amount of money won during the year (in dollars) and the average number of strokes above 70 for each player in the sample were recorded. Lower scoring averages are better in golf.

Adjusted scoring average	Earnings ($)	Adjusted scoring average	Earnings ($)	Adjusted scoring average	Earnings ($)
4.88	22,927	4.10	36,403	1.02	448,048
3.57	44,332	2.78	121,347	2.98	70,790
1.86	459,449	1.38	636,734	4.50	17,944
1.87	104,603	2.79	212,352	0.16	1,626,297
3.41	122,540	2.08	149,872		

$y - \hat{y}$

a.) Calculate an exponential model for these data using adjusted scoring average as the explanatory variable.

$$\hat{y} = 1479695.732 \times 0.405^x$$

b.) Calculate and interpret the residual for Lorena Ochoa, the last golfer in this list.

$$\hat{y} = 1479695.732 \times (0.405)^{0.16}$$

$$\hat{y} = 1280457.667$$

$$y - \hat{y} = 1626297 - 1280457.667$$

$$= 345839.3351$$

The actual earnings are about $345839 more than the predicted value.