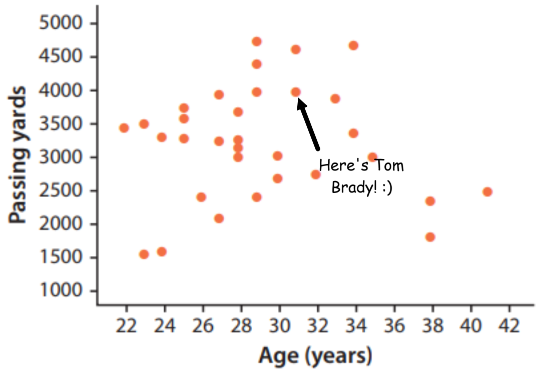
2.8 Fitting Models to Curved Relationships Part 1: Quadratic Models

For linear association, we can use a least-squares regression line to model the relationship

We can use a **quadratic model** to fit a nonlinear association between two quantitative variables.

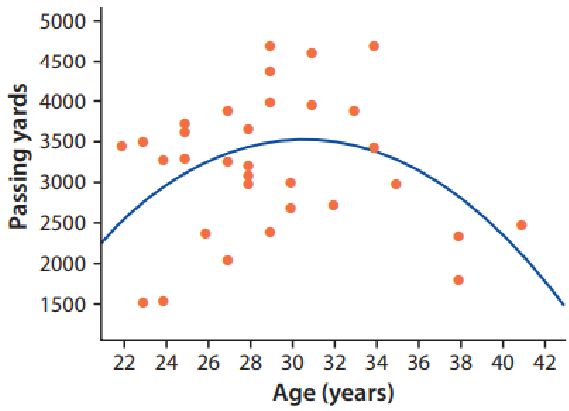
A **quadratic model** is a model in the form $\hat{y} = ax^2 + bx + c$. The graph of a quadratic model is a parabola.



The scatterplot here shows the relationship between passing yards and age for 32 quarterbacks in the 2010 NFL season.

The association between passing yards & age is clearly nonlinear. Passing yards tend to be lower for younger and older quarterbacks and higher for "middle-aged" quarterbacks.

The same scatterplot here includes the quadratic model:
$\hat{y} = -13.65x^2 + 835.5x - 9258$
The quadratic model fits the form of this association quite well.



Brett Farve has 41 years old in 2010 and had 2,509 passing yards. The quadratic model suggests that he would have:
$\hat{y} = -13.65(41)^2 + 835.5(41) - 9258 = 2052$ yards
This means that Farve had 2509 - 2052 = 457 more passing yards than predicted, based on the quadratic model using x = age.
In other words, the residual for Brett Farve is 457 yards.
We interpret r^2 and s in the same way for a linear model: 17.6 % of the variability in passing yards is accounted for by the quadratic model using x = age. Also, when using the quadratic model with x = age, our predictions of passing yards are typically off by about 802 yards.

__Example__: How is the braking distance for a motorcycle related to the speed the motorcycle was going when the brake was applied? Aaron Waggoner gathered data to answer this question. The table shows the speed (in miles per hour) and the distance needed to come to a complete stop when the break was applied (in feet).

Speed (mph)	Distance (ft)	Speed (mph)	Distance (ft)
6	1.42	32	52.08
9	4.92	40	84.00
19	18.00	48	110.33
30	44.75		

a.) Calculate a quadratic model for these data using speed as the explanatory variable.

$$\hat{y} = -2.294 + 0.33x + 0.043x^2$$

b.) Calculate and interpret the residual for the last observation in the table.

$$\hat{y} = -2.294 + 0.33(48) + 0.043(48)^2$$

$$\hat{y} = 112.618$$

$$y - \hat{y} = 110.33 - 112.618 = -2.288$$

The actual braking distance will be about 2.288 away from the prediction.

c.) Sketch the scatterplot, along with the quadratic model.