8.5 Properties of Logarithms

Product Property $\log_b m + \log_b n = \log_b mn$

Quotient Property $\log_b m - \log_b n = \log_b \frac{m}{n}$

Power Property $n \log_b m = \log_b m^n$

Properties of Logarithms

1. $\log_b 1 = 0 \Rightarrow b^0 = 1$
2. $\log_b b = 1 \Rightarrow b^1 = b$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

One-to-One Property of Logarithms

If $\log_b x = \log_b y$, then $x = y$.

Examples

Given $\log_2 3 \approx 1.5850$, approximate the value of each expression below.

1. $\log_2 12$ $= \log_2 4 \cdot 3$

$\log_2 4 + \log_2 3$

$\log_2 2^2 + 1.5850$

$2 + 1.5850$

$\boxed{3.5850}$

2. $\log_2 1.5 = \log_2 3 \cdot 0.5$

$\log_2 3 + \log_2 \frac{1}{2}$

$1.5850 + \log_2 2^{-1}$

$1.5850 + -1$

$\boxed{0.5850}$

Given $\log_3 7 \approx 1.7712$, approximate the value of each expression below.

3. $\log_3 \frac{3}{7}$

$\log_3 3^1 - \log_3 7$

$1 - 1.7712$

$\boxed{-0.7712}$

4. $\log_3 49 = \log_3 7 \cdot 7$

$\log_3 7 + \log_3 7$

$1.7712 + 1.7712$

$\boxed{3.5424}$

Examples

Write each expression as a single logarithm. Then simplify, if possible.

5. $\log_2 5 + \log_2 7$

$\log_2 5 \cdot 7 = \boxed{\log_2 35}$

6. $\log_3 45 - \log_3 9$

$\log_3 \frac{45}{9} = \boxed{\log_3 5}$

7. $\log_2 5 + \log_2 x - \log_2 10$

$\log_2 5 \cdot x$

$\log_2 5x - \log_2 10$

$\log_2 \frac{5x}{10} = \boxed{\log_2 \frac{x}{2}}$

8. $\log_7 3x - \log_7 9x + \log_7 6y$

$\log_7 \frac{3x}{9x}$

$\log_7 \frac{1}{3} + \log_7 6y$

$\log_7 \frac{1}{3} \cdot 6y = \boxed{\log_7 2y}$

9. $5 \log_2 m - 2 \log_2 n$

$\log_2 m^5 - \log_2 n^2$

$\log_2 \frac{m^5}{n^2}$

10. $4 \log_b m + \log_b n - \frac{1}{2} \log_b p$

$\log_b m^4 + \log_b n - \log_b p^{1/2}$

$\log_b m^4 n - \log_b p^{1/2}$

$\boxed{\log_b \frac{m^4 n}{p^{1/2}}}$

Examples

Write each expression as a sum or difference of logarithms. Then simplify, if possible.

11. $\log_2 4x^3$

$\log_2 4 + \log_2 x^3$

$\log_2 2^2 + 3\log_2 x$

$2 + 3\log_2 x$

12. $\log_4 \frac{4a}{b^2}$

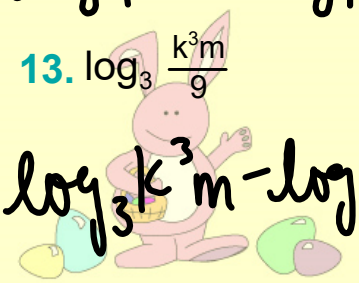
$\log_4 4a - \log_4 b^2 = \log_4 4 + \log_4 a - \log_4 b^2$

$1 + \log_4 a - 2\log_4 b$

13. $\log_3 \frac{k^3 m}{9}$

$\log_3 k^3 m - \log_3 9 = \log_3 k^3 + \log_3 m - \log_3 3^2$

$3\log_3 k + \log_3 m - 2$



The calculator only uses a base of 10 for logarithms. So when we have a logarithm with any other base, we need another way to evaluate it.

CHANGE OF BASE FORMULA

For any positive real numbers a ≠ 1, b ≠ 1, and x > 0:

$$\log_b x = \frac{\log_a x}{\log_a b}$$

Examples

Evaluate each.

14. $\log_7 32$

15. $\log_{15} 6$

16. $\log_{\frac{1}{4}} 20$