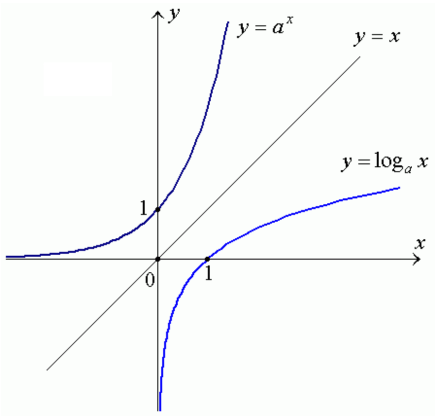
Since the log function is the inverse of the exponential function, it can be graphed by switching the domain and range.

Since $f(x) = a^x$ is a rapidly increasing function, $f(x) = \log_a x$ is a very slowly increasing function.



Notice...

1) that since $a^0 = 1$, then $\log_a 1 = 0$.

2) that since the x-axis is the asymptote for the exponential function, then the y-axis is the asymptote for the log function (unless there is a shift).

3) that the log function is a reflection across the line <u>y = x</u>.

Example 6

Sketch the graph by making a table and plotting points. State the domain, range, and any asymptotes.

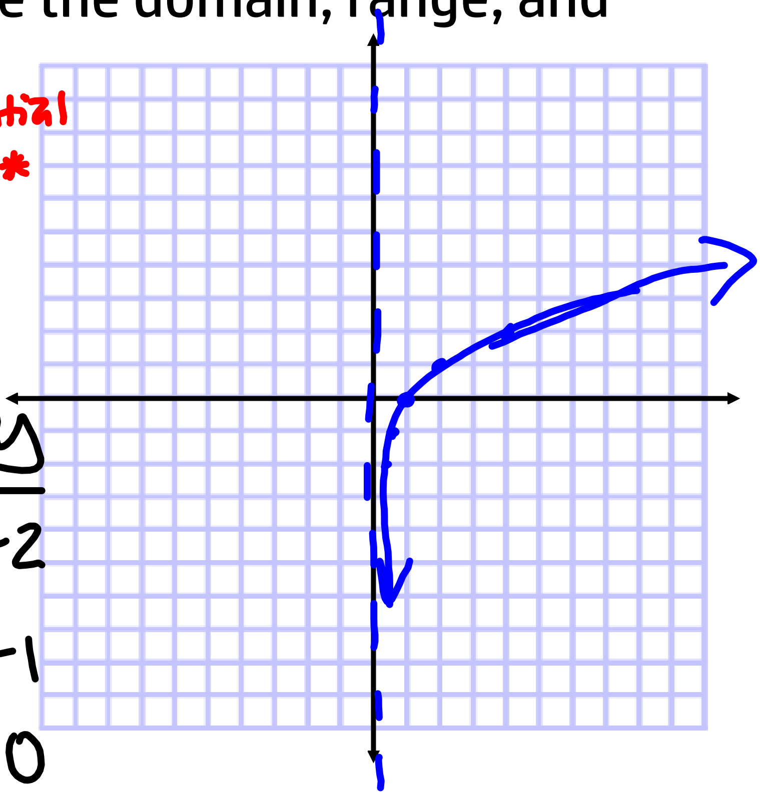
rewrite in exponential form

$y = \log_2 x$

$$\Downarrow$$

$$2^y = x$$

x		y
$\frac{1}{4}$	$2^{-2} = \frac{1}{2^2}$	-2
$\frac{1}{2}$	$2^{-1} = \frac{1}{2^1}$	-1
1	2^0	0
2	2^1	1
4	2^2	2



Example 7

Sketch the graph by plotting points.
State the domain, range, and any asymptotes.

$y = 2 + \log_2(x - 3)$

$-2 \qquad -2$

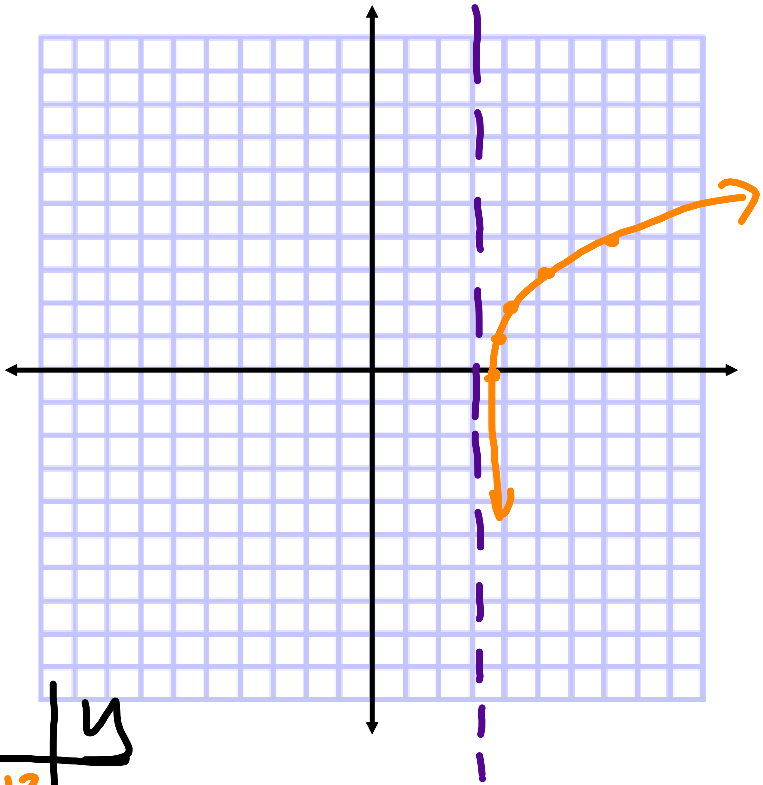
$y - 2 = \log_2(x-3)$

$\Downarrow$

$2^{y-2} = x - 3$

$+3 \qquad +3$

$2^{\boxed{y-2}} \boxed{+3} = x$



x		y
$\frac{13}{4} = 3\frac{1}{4}$	$2^{0-2}+3 = 2^{-2}+\frac{3}{1} = \frac{1}{4}+\frac{12}{4}$	0
$\frac{7}{2} = 3\frac{1}{2}$	$2^{1-2}+3 = 2^{-1}+3 = \frac{1}{2}+\frac{6}{2}$	1
4	$2^{2-2}+3 = 2^{0}+3 = 1+3$	2
5	$2^{3-2}+3 = 2^{1}+3 = 2+3$	3
7	$2^{4-2}+3 = 2^{2}+3 = 4+3$	4

$y-2=-2$, $+2 \quad +2$, $y=0$

$y-2=-1$, $+2 \quad +2$, $y=1$

$y-2=0$, $+2 \quad +2$, $y=2$

$y-2=1$, $+2 \quad +2$, $y=3$

$y-2=2$, $+2 \quad +2$, $y=4$

Example 8

Sketch the graph by plotting points.
State the domain, range, and any asymptotes.

$$\frac{y}{-1} = \frac{-\log_2 x}{-1}$$

$$-y = \log_2 x$$

$$\Downarrow$$

$$2^{-y} = x$$

x		y
4	$2^{-(-2)} = 2^2 = 4$	-2
2	$2^{-(-1)} = 2^1 = 2$	-1
1	$2^{-0} = 2^0 = 1$	0
$\frac{1}{2}$	$2^{-1} = \frac{1}{2^1} = \frac{1}{2}$	1
$\frac{1}{4}$	$2^{-2} = \frac{1}{2^2} = \frac{1}{4}$	2