

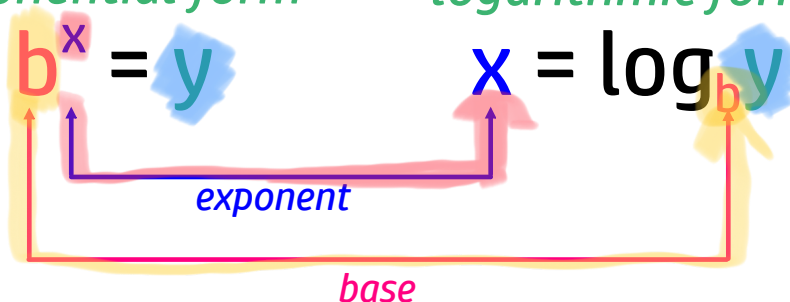
8.4 Logarithmic Functions

Every **exponential function** $y = b^x$ with $b > 0$ and $b \neq 1$ passes the horizontal line test.

Therefore it has an **inverse function**, which is called the **logarithmic function**.

exponential form

logarithmic form



"log base b of y is x"

Example 1 $b^x = y \Rightarrow x = \log_b y$

Rewrite each exponential in logarithmic form.

a) $10^5 = 100,000$ $5 = \log_{10} 100,000$

↑
base

b) $2^3 = 8$ $3 = \log_2 8$

↑
base

c) $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ $2 = \log_{\frac{1}{3}} \frac{1}{9}$

↑
base

Example 2 $x = \log_b y \Rightarrow b^x = y$

Rewrite each logarithm in exponential form.

a) $\log_2 32 = 5$ $2^5 = 32$

b) $\log_5 \left(\frac{1}{25}\right) = -2$ $5^{-2} = \frac{1}{25}$

c) $\log_7 h = g$ $7^g = h$

It is **important** to remember
that $\log_a x$ is an exponent!
The answer to every log is the exponent!

One-to-One Property of Exponents

$$\text{If } b^x = b^y, \text{ then } x = y.$$

Example 3 $\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_5 25 = x$ $5^x = 25 \Rightarrow 5^x = 5^2$ $x=2$

b) $\log_{10} 1 = x$ $10^x = 1 \Rightarrow 10^x = 10^0$ $x=0$

c) $\log_3 81 = x$ $3^x = 81 \Rightarrow 3^x = 3^4$ $x=4$

d) $\log_8 \frac{1}{64} = x$ $8^x = \frac{1}{64} \Rightarrow 8^x = 64^{-1} \Rightarrow 8^x = 8^{-2}$
 $8^{2 \cdot -1} = 8^{-2}$ $x=-2$

Example 4 $\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_x 49 = 2$ $x^2 = 49 \Rightarrow x^2 = 7^2$ $x=7$

b) $\log_x \frac{1}{16} = -4$ $x^{-4} = \frac{1}{16} \Rightarrow x^{-4} = 16^{-1} \Rightarrow x^{-4} = 2^{-4}$ $x=2$

c) $\log_x \frac{1}{125} = 3$ $x^3 = \frac{1}{125} \Rightarrow x^3 = \left(\frac{1}{5}\right)^3$ $x = \frac{1}{5}$

d) $\log_x 64 = 1$ $x^1 = 64 \Rightarrow x=64$

Example 5 $\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_3 x = -3$ $3^{-3} = x \Rightarrow x = \frac{1}{3^3} \Rightarrow \boxed{x = \frac{1}{27}}$

b) $\log_5 x = 4$ $5^4 = x \Rightarrow x = \underbrace{5 \cdot 5}_{25} \cdot \underbrace{5 \cdot 5}_{25} \Rightarrow \boxed{x = 625}$

c) $\log_{27} x = \frac{1}{3}$ $27^{1/3} = x \Rightarrow x = (\sqrt[3]{27})^1 \Rightarrow x = 3^1 \Rightarrow \boxed{x = 3}$

d) $\log_{10} x = 0$ $10^0 = x \Rightarrow \boxed{x = 1}$