8.4 Logarithmic Functions

Every exponential function $y = b^x$ with $b > 0$ and $b \neq 1$ passes the horizontal line test. Therefore it has an inverse function, which is called the logarithmic function.

exponential form	*logarithmic form*
$b^x = y$	$x = \log_b y$

exponent

base

"log base b of y is x"

Example 1

$b^x = y \Rightarrow x = \log_b y$

Rewrite each exponential in logarithmic form.

a) $10^5 = 100{,}000$ exp

base

$5 = \log_{10} 100000$

b) $2^3 = 8$ exp.

base

$3 = \log_2 8$

c) $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ exp.

base

$2 = \log_{1/3} \frac{1}{9}$

Example 2

$x = \log_b y \Rightarrow b^x = y$

Rewrite each logarithm in exponential form.

a) $\log_2 32 = 5$ $2^5 = 32$

b) $\log_5 \left(\frac{1}{25}\right) = -2$ $5^{-2} = \frac{1}{25}$

c) $\log_7 h = g$ $7^g = h$

It is **important** to remember
that $\log_a x$ is an exponent!
The answer to every log is the exponent!

One-to-One Property of Exponents

If $b^x = b^y$, then $x = y$.

Example 3

$\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_5 25 = x$

$5^x = 25 \Rightarrow 5^x = 5^2$ $\boxed{x=2}$

b) $\log_{10} 1 = x$

$10^x = 1 \Rightarrow 10^x = 10^0$ $\boxed{x=0}$

c) $\log_3 81 = x$

$3^x = 81 \Rightarrow 3^x = 3^4$ $\boxed{x=4}$

d) $\log_8 \frac{1}{64} = x$

$8^x = \frac{1}{64} \Rightarrow 8^x = 64^{-1} \Rightarrow 8^x = 8^{-2}$

$8^{2 \cdot -1} = 8^{-2}$ $\boxed{x=-2}$

Example 4

$\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_x 49 = 2$

$x^2 = 49 \Rightarrow x^2 = 7^2$ $\boxed{x=7}$

b) $\log_x \frac{1}{16} = -4$

$x^{-4} = \frac{1}{16} \Rightarrow x^{-4} = 16^{-1} \Rightarrow x^{-4} = 2^{-4}$ $\boxed{x=2}$

c) $\log_x \frac{1}{125} = 3$

$x^3 = \frac{1}{125} \Rightarrow x^3 = \left(\frac{1}{5}\right)^3$ $\boxed{x=\frac{1}{5}}$

d) $\log_x 64 = 1$

$x^1 = 64 \Rightarrow \boxed{x=64}$

Example 5 $\log_b y = x \Rightarrow b^x = y$

Evaluate using the properties of logs.

a) $\log_3 x = -3$ $\quad 3^{-3} = x \Rightarrow x = \frac{1}{3^3} \Rightarrow \boxed{x = \frac{1}{27}}$

b) $\log_5 x = 4$ $\quad 5^4 = x \Rightarrow x = \underbrace{5 \cdot 5}_{25} \cdot \underbrace{5 \cdot 5}_{25} \Rightarrow \boxed{x = 625}$

c) $\log_{27} x = \frac{1}{3}$ $\quad 27^{1/3} = x \Rightarrow x = \left(\sqrt[3]{27}\right)^1 \Rightarrow x = 3^1 \Rightarrow \boxed{x = 3}$

d) $\log_{10} x = 0$

$10^0 = x \Rightarrow \boxed{x = 1}$