POWER FUNCTIONS,

FUNCTION OPERATIONS,

&

COMPOSITION OF FUNCTIONS

Suppose: $f(x) = 2x$ and $g(x) = x + 1$

Addition $f(x) + g(x)$
$= (2x) + (x + 1)$
$= 3x + 1$

Subtraction $f(x) - g(x)$
$= (2x) - (x + 1)$
$= 2x - x - 1$
$= x - 1$

Multiplication $f(x) \bullet g(x)$
$= 2x(x+1)$
$= 2x^2 + 2x$

Division $\frac{f(x)}{g(x)}$ $\frac{2x}{x + 1}$

So far we have learned...

- linear functions
- quadratic functions
- functions of higher degrees

power function: in the form of $y = ax^b$

(a is a real number and b is a rational number)

Ex: $y = 2x^{\frac{1}{2}}$

Example 1: Let $f(x) = 2x^{\frac{1}{2}}$ and $g(x) = -6x^{\frac{1}{2}}$.

Find the (a) $f(x) + g(x)$, and state the domain
(b) $f(x) - g(x)$, and state the domain
(c) $f(x) \cdot g(x)$, and state the domain

Example 2: Let $f(x) = 3x^{\frac{1}{3}}$ and $g(x) = 2x^{\frac{1}{3}}$.

Find the (a) $f(x) + g(x)$, and state the domain
(b) $g(x) - f(x)$, and state the domain
(c) $f(x) \cdot g(x)$, and state the domain

Example 3: Let $f(x) = -5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}$ and
$g(x) = 8x^{\frac{1}{2}} + 9x^{\frac{1}{3}}$.

Find the (a) $f(x) + g(x)$, and state the domain
(b) $f(x) - g(x)$, and state the domain
(c) $f(x) + f(x)$, and state the domain
(d) $g(x) - g(x)$, and state the domain

Example 4: Let $f(x) = 8x^{\frac{1}{3}}$ and $g(x) = 4x^{\frac{3}{2}}$.

Find the (a) $f(x) \cdot g(x)$, and state the domain

(b) $\frac{f(x)}{g(x)}$, and state the domain

(c) $\frac{g(x)}{f(x)}$, and state the domain

COMPOSITION OF FUNCTIONS

Example 5: Let $f(x) = 5x^{-1}$, $g(x) = 3x - 1$, and $h(x) = \frac{x+4}{6}$.

Find the (a) $f(g(x))$

(b) $g(h(x))$

(c) $f(f(x))$

(a) $f(g(x)) = f(3x-1) = 5(3x-1)^{-1} = \boxed{\frac{5}{3x-1}}$

(b) $g(h(x)) = g\left(\frac{x+4}{6}\right) = \frac{3}{1}\left(\frac{x+4}{6}\right) - 1 = \boxed{\frac{x+4}{2} - 1}$

(c) $f(f(x)) = f(5x^{-1}) = 5(5x^{-1})^{-1} = \frac{5}{5x^{-1}} = \frac{1}{x^{-1}} = \boxed{x}$

Example 6: Let $f(x) = 2x^{-1}$, $g(x) = x^2 - 1$, and $h(x) = \frac{x - 5}{4}$.

Find the (a) $g(f(x))$
(b) $g(g(x))$
(c) $h(g(x))$

(a) $g(f(x)) = g(2x^{-1}) = (2x^{-1})^2 - 1 = 4x^{-2} - 1 = \boxed{\frac{4}{x^2} - 1}$

(b) $g(g(x)) = g(x^2-1) = (x^2-1)^2 - 1$
$= (x^2-1)(x^2-1) - 1$
$= x^4 - x^2 - x^2 + 1 - 1 = \boxed{x^4 - 2x^2}$

(c) $h(g(x)) = h(x^2-1) = \frac{x^2-1-5}{4} = \boxed{\frac{x^2-6}{4}}$

Example 7: Let $f(x) = 4x^{-1}$, $g(x) = 3x - 3$ and $h(x) = \frac{x+3}{8}$.

(a) $h(f(2)) = h(2)$

$f(2) = 4(2)^{-1}$
$= \frac{4}{2} = 2$

$h(2) = \frac{2+3}{8} = \boxed{\frac{5}{8}}$

(b) $f(g(-3)) = f(-12)$

$g(-3) = 3(-3) - 3$
$= -9 - 3 = -12$

$f(-12) = 4(-12)^{-1}$
$= \frac{4}{-12}$
$= \boxed{\frac{1}{-3}}$

(c) $g(h(21)) = g(3)$

$h(21) = \frac{21+3}{8} = \frac{24}{8} = 3$

$g(3) = 3(3) - 3$
$= 9 - 3$
$= \boxed{6}$