POWER FUNCTIONS,

FUNCTION OPERATIONS,

&

COMPOSITION OF FUNCTIONS

Suppose: $f(x) = 2x$ and $g(x) = x + 1$

Addition $f(x) + g(x) = (2x) + (x + 1)$
$= 3x + 1$

Subtraction $f(x) - g(x) = (2x) - (x + 1)$
$= 2x - x - 1$
$= x - 1$

Multiplication $f(x) \cdot g(x) = 2x(x+1)$
$= 2x^2 + 2x$

Division $\dfrac{f(x)}{g(x)} \quad \dfrac{2x}{x + 1}$

So far we have learned...

- linear functions
- quadratic functions
- functions of higher degrees

power function: in the form of $y = ax^b$

(a is a real number and b is a rational number)

fraction

Ex: $y = 2x^{\frac{1}{2}}$

Example 1: Let $f(x) = 2x^{\frac{1}{2}}$ and $g(x) = -6x^{\frac{1}{2}}$.

Find the (a) $f(x) + g(x)$, ~~and state the domain~~

(b) $f(x) - g(x)$, ~~and state the domain~~

(c) $f(x) \cdot g(x)$, ~~and state the domain~~

(a) $f(x) + g(x) = 2x^{\frac{1}{2}} + -6x^{1/2} = \boxed{-4x^{\frac{1}{2}}}$

(b) $f(x) - g(x) = 2x^{1/2} + 6x^{\frac{1}{2}} = 2x^{\frac{1}{2}} + 6x^{\frac{1}{2}} = \boxed{8x^{\frac{1}{2}}}$

(c) $f(x) \cdot g(x) = (2x^{\frac{1}{2}})(-6x^{\frac{1}{2}}) = -12x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$

$= -12x^{\frac{1}{2}+\frac{1}{2}} = \boxed{-12x}$

Example 2: Let $f(x) = 3x^{\frac{1}{3}}$ and $g(x) = 2x^{\frac{1}{3}}$.

Find the (a) $f(x) + g(x)$, ~~and state the domain~~
(b) $g(x) - f(x)$, ~~and state the domain~~
(c) $f(x) \cdot g(x)$, ~~and state the domain~~

ⓐ $f(x)+g(x) = 3x^{\frac{1}{3}} + 2x^{\frac{1}{3}} = \boxed{5x^{\frac{1}{3}}}$

ⓑ $g(x)-f(x) = 2x^{\frac{1}{3}} - 3x^{\frac{1}{3}} = \boxed{-1x^{\frac{1}{3}} \text{ or } -x^{\frac{1}{3}}}$

ⓒ $f(x)\cdot g(x) = (3x^{\frac{1}{3}})(2x^{\frac{1}{3}}) = 6x^{\frac{1}{3}}\cdot x^{\frac{1}{3}} = \boxed{6x^{\frac{2}{3}}}$

Example 3: Let $f(x) = -5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}$ and
$g(x) = 8x^{\frac{1}{2}} + 9x^{\frac{1}{3}}$.

Find the (a) $f(x) + g(x)$, ~~and state the domain~~
(b) $f(x) - g(x)$, ~~and state the domain~~
(c) $f(x) + f(x)$, ~~and state the domain~~
(d) $g(x) - g(x)$, ~~and state the domain~~

ⓐ $f(x)+g(x) = (-5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}) + (8x^{\frac{1}{2}} + 9x^{\frac{1}{3}}) = \boxed{3x^{\frac{1}{2}} + 14x^{\frac{1}{3}}}$

ⓑ $f(x)-g(x) = (-5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}) - (8x^{\frac{1}{2}} + 9x^{\frac{1}{3}})$
$-5x^{\frac{1}{2}} + 5x^{\frac{1}{3}} - 8x^{\frac{1}{2}} - 9x^{\frac{1}{3}} = \boxed{-13x^{\frac{1}{2}} - 4x^{\frac{1}{3}}}$

ⓒ $f(x)+f(x) = (-5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}) + (-5x^{\frac{1}{2}} + 5x^{\frac{1}{3}}) = \boxed{-10x^{\frac{1}{2}} + 10x^{\frac{1}{3}}}$

ⓓ $g(x)-g(x) = (8x^{\frac{1}{2}} + 9x^{\frac{1}{3}}) - (8x^{\frac{1}{2}} + 9x^{\frac{1}{3}})$
$8x^{\frac{1}{2}} + 9x^{\frac{1}{3}} - 8x^{\frac{1}{2}} - 9x^{1/3} = 0 + 0 = \boxed{0}$

Example 4: Let $f(x) = 8x^{\frac{1}{3}}$ and $g(x) = 4x^{\frac{3}{2}}$.

Find the (a) $f(x)\cdot g(x)$, ~~and state the domain~~

(b) $\frac{f(x)}{g(x)}$, ~~and state the domain~~

(c) $\frac{g(x)}{f(x)}$, ~~and state the domain~~

ⓐ $f(x)\cdot g(x) = \left(8x^{\frac{1}{3}}\right)\left(4x^{\frac{3}{2}}\right) = 32x^{\frac{1\cdot 2}{3\cdot 2}+\frac{3\cdot 3}{2\cdot 3}} = 32x^{\frac{2}{6}+\frac{9}{6}}$

$= \boxed{32x^{11/6}}$

ⓑ $\frac{f(x)}{g(x)} = \frac{8x^{\frac{1}{3}}}{4x^{3/2}} = 2x^{\frac{1\cdot 2}{3\cdot 2}-\frac{3\cdot 3}{2\cdot 3}} = 2x^{\frac{2}{6}-\frac{9}{6}} = 2x^{-7/6} = \boxed{\frac{2}{x^{7/6}}}$

ⓒ $\frac{g(x)}{f(x)} = \frac{4x^{3/2}}{8x^{1/3}} = \frac{1}{2}x^{\frac{3\cdot 3}{2\cdot 3}-\frac{1\cdot 2}{3\cdot 2}} = \frac{1}{2}x^{\frac{9}{6}-\frac{2}{6}} = \boxed{\frac{1}{2}x^{7/6} \text{ or } \frac{x^{7/6}}{2}}$

COMPOSITION OF FUNCTIONS

Example 5: Let $f(x) = 5x^{-1}$, $g(x) = 3x - 1$, and $h(x) = \frac{x+4}{6}$.

Find the (a) $f(g(x))$

(b) $g(h(x))$

(c) $f(f(x))$

Example 6: Let $f(x) = 2x^{-1}$, $g(x) = x^2 - 1$, and $h(x) = \frac{x-5}{4}$.

Find the (a) $g(f(x))$
(b) $g(g(x))$
(c) $h(g(x))$

Example 7: Let $f(x) = 4x^{-1}$, $g(x) = 3x - 3$ and $h(x) = \frac{x+3}{8}$.

(a) $h(f(2))$ (b) $f(g(-3))$ (c) $g(h(21))$