1.9 Describing Location in a Distribution (Part 3)

A percentile is one way to describe the location of an individual in a distribution of quantitative data. Another way is to give the **standardized score** (z-score) for the observed value.

The **standardized score (z-score)** for an individual value in a distribution tells us how many standard deviations from the mean the value falls, and in what direction.
To find the standardized score (z-score), compute

$$z = \frac{\text{value - mean}}{\text{standard deviation}}$$

Values larger than the mean have positive z-scores. Values smaller than the mean have negative z-scores.

Example: Women's heights have a mean of 63.6 in. and a standard deviation of 2.5 inches. Find the z-score corresponding to a woman with a height of 70 inches and determine whether the height is unusual.

$$Z = \frac{\text{value} - \text{mean}}{S.D.}$$

$$Z = \frac{70 - 63.6}{2.5} = \frac{6.4}{2.5}$$

$$\boxed{Z = 2.56}$$

Example: Answer the following questions regarding z-scores.

a.) Mr. Pryor's first period statistics test had a mean of 80 and a standard deviation of 6.07. Find the z-score for Lionel, who earned a 67 on the test.

$$z = \frac{value - mean}{S.D.} = \frac{67 - 80}{6.07} = \frac{-13}{6.07}$$

$$= -2.14168\ldots \approx \boxed{-2.14}$$

b.) The z-score for Alexa is 1.3. Find the grade that she made on the test.

$$z = \frac{value - mean}{S.D.}$$

$$6.07 \cdot 1.3 = \frac{value - 80}{6.07} \cdot 6.07$$

$$7.891 = value - 80$$
$$+80 \qquad\qquad +80$$
$$value = 87.891$$

Alexa made an 88 on the test.

We often standardize observed values to express them on a common scale. For example, we might compare the heights of two children of different ages or genders by calculating their z-scores.

- At age 7, Jordan is 51 inches tall. Her height puts her at a z-score of 1. That is, Jordan is 1 standard deviation above the mean height of 7-year-old girls.

- Zayne's height at age 9 is 54 inches. His corresponding z-score is 0.5. In other words, Zayne is one-half standard deviation above the mean of 9-year-old boys.

So Jordan is taller relative to girls her age than Zayne is relative to boys his age. The standardized heights tell where each child stands (pun intended!) in the distribution for his or her age group.