7.1 nth Roots and Rational Exponents

KEY CONCEPT *For Your Notebook*

Properties of Exponents REMINDER

Let a and b be real numbers and let m and n be integers.

Property Name	Definition	Example
Product of Powers	$a^m \cdot a^n = a^{m+n}$	$5^3 \cdot 5^{-1} = 5^{3+(-1)} = 5^2 = 25$
Power of a Power	$(a^m)^n = a^{mn}$	$(3^3)^2 = 3^{3 \cdot 2} = 3^6 = 729$
Power of a Product	$(ab)^m = a^m b^m$	$(2 \cdot 3)^4 = 2^4 \cdot 3^4 = 1296$
Negative Exponent	$a^{-m} = \frac{1}{a^m}, a \neq 0$	$7^{-2} = \frac{1}{7^2} = \frac{1}{49}$
Zero Exponent	$a^0 = 1, a \neq 0$	$(-89)^0 = 1$
Quotient of Powers	$\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6^{-3}}{6^{-6}} = 6^{-3-(-6)} = 6^3 = 216$
Power of a Quotient	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{4}{7}\right)^2 = \frac{4^2}{7^2} = \frac{16}{49}$

Rational Exponents

A radical expression contains a radical symbol and index number or a rational (fraction) exponent.

even # roots can only take positives

odd # roots can be negative inside

Radical Notation

$\sqrt{32x^5}$ square root

The only one without an index number.

$\sqrt[3]{64y^6}$ cube root

$\sqrt[4]{16x^8}$ fourth root

$\sqrt[5]{64}$ fifth root

radicands

Exponential Notation

a. $\sqrt[n]{a} = a^{\frac{1}{n}}$ (index number becomes the denominator on the exponent)

b. $\sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m = a^{\frac{m}{n}}$ (exponent becomes the numerator)

c. $a^{-\frac{m}{n}} = \frac{1}{\sqrt[n]{a^m}}$

Rewrite using rational exponent notation.

1. $\sqrt{21}$ → $21^{\frac{1}{2}}$

2. $\sqrt[3]{25}$ → $25^{\frac{1}{3}}$

3. $(\sqrt[4]{36})^3$ → $36^{\frac{3}{4}}$

4. $(\sqrt[3]{6})^2$ → $6^{\frac{2}{3}}$

5. $\sqrt[4]{15}$ → $15^{\frac{1}{4}}$

6. $(\sqrt{7})^5$ → $7^{\frac{5}{2}}$

Rewrite using radical notation.

7. $5^{\frac{5}{6}}$

$\sqrt[6]{5^5}$ or $(\sqrt[6]{5})^5$

8. $21^{\frac{3}{2}}$

$\sqrt{21^3}$ or $(\sqrt{21})^3$

9. $10^{-\frac{2}{3}} = \frac{1}{10^{2/3}} = \frac{1}{\sqrt[3]{10^2}}$

10. $9^{\frac{4}{3}}$

$\sqrt[3]{9^4}$

11. $8^{-\frac{2}{5}}$

$\frac{1}{8^{2/5}} = \frac{1}{\sqrt[5]{8^2}}$

12. $12^{\frac{1}{4}}$

$\sqrt[4]{12^1}$

Evaluate.

13. $\sqrt[3]{-8}$

$\sqrt[3]{-2 \cdot -2 \cdot -2}$

-2

14. $(\sqrt[4]{81})^3$

$(3)^3$

27

15. $(\sqrt{25})^3$

$(5)^3$

125

16. $(\sqrt[3]{-27})^{-2}$

$(-3)^{-2}$

$\frac{1}{(-3)^2}$

$\frac{1}{9}$

17. $(\sqrt[4]{16})^{-1}$

$(2)^{-1}$

$\frac{1}{2^1}$

$\frac{1}{2}$

18. $(\sqrt{49})^{-3}$

$(7)^{-3}$

$\frac{1}{7^3}$

$\frac{1}{343}$

Evaluate.

19. $256^{\frac{1}{4}}$

$\sqrt[4]{256}$

4

20. $27^{\frac{4}{3}}$

$(\sqrt[3]{27})^4$

$(3)^4 = 81$

21. $32^{-\frac{2}{5}} = \frac{1}{32^{\frac{2}{5}}}$

$\frac{1}{(\sqrt[5]{32})^2} = \frac{1}{(2)^2} = \frac{1}{4}$

22. $64^{\frac{5}{6}}$

$(\sqrt[6]{64})^5$

$(2)^5 = 32$

23. $36^{\frac{3}{2}}$

24. $216^{-\frac{2}{3}}$

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Solve each equation.

25. $x^3 = 125$

26. $-2x^4 = -32$

27. $\frac{1}{2}x^3 + 6 = 10$

28. $(x + 2)^3 = 27$

29. $(x - 5)^4 = 256$

30. $-x^5 + 3 = 35$