

8.1 Solving Equations with Rational Coefficients

An equation is a mathematical sentence stating that two expressions are equal.

A value for the variable that makes an equation true is called a solution.

For example, if 72 teens in a survey say they have a social network profile, and these teens are three fourths of the teens surveyed, then the equation $\frac{3}{4}x = 72$ can be used to find the total number of teens surveyed. For this equation, the solution is 96, since $\frac{3}{4}(96) = 72$ is a true statement.

You can use equality properties and inverse operations to solve an equation.

Inverse operations "undo" each other. To undo the multiplication in the previous example, you can divide each side of the equation by $\frac{3}{4}$.

Applying the equality properties creates equivalent equations, which are equations that have the same solution.

Example: Solve each equation.

$$\text{a.) } 4.2x = -52.5$$

$$\div 4.2 \quad \div 4.2$$

$$x = -52.5 \div 4.2$$

$$x = -12.5$$

$$\begin{array}{r} 12.5 \\ 4.2 \overline{) 52.50} \\ \underline{-42} \\ 105 \\ \underline{-84} \\ 210 \\ \underline{-210} \\ 0 \end{array}$$

$$\text{b.) } 5.6x = -19.6$$

$$\div 5.6 \quad \div 5.6$$

$$x = -19.6 \div 5.6$$

$$x = -3.5$$

$$\begin{array}{r} 3.5 \\ 5.6 \overline{) 19.60} \\ \underline{-168} \\ 280 \\ \underline{-280} \\ 0 \end{array}$$

$$\text{c.) } 7.4x = -62.9$$

$$\div 7.4 \quad \div 7.4$$

$$x = -62.9 \div 7.4$$

$$x = -8.5$$

$$\begin{array}{r} 8.5 \\ 7.4 \overline{) 62.90} \\ \underline{-592} \\ 370 \\ \underline{-370} \\ 0 \end{array}$$

Example: A one-year, in-state camping permit for New Mexico State Parks costs \$180. If the total income from the camping permits is \$8,280 during the first day of sales, write and solve an equation to find how many permits were purchased.

Let p be the permits purchased.

$$180p = 8280$$

$$\div 180 \quad \div 180$$

$$p = 8280 \div 180 = 46$$

$$\begin{array}{r} 46 \\ 180 \overline{) 8280} \\ \underline{-72} \\ 108 \\ \underline{-108} \\ 0 \end{array}$$

46 permits were bought.

Example: Esteban spent \$116 on boxes of baseball cards. He paid \$14.50 per box. Write and solve an equation to find how many boxes of cards Esteban bought.

Let c be the boxes of baseball cards.

$$14.50c = 116$$

$$c = 116 \div 14.5 = 8$$

Esteban bought 8 boxes of cards.

$$\begin{array}{r} 34 \\ 145 \\ \times 8 \\ \hline 1160 \end{array}$$

$$\begin{array}{r} 8 \\ 14.5 \overline{) 116.0} \\ \underline{-1160} \\ 0 \end{array}$$

Equations in which a variable has a fraction as a coefficient can be solved by multiplying the "flip" instead of dividing the fraction.

Example: Solve each equation.

a.) $\frac{1}{4}y = -8$

$$y = -8 \cdot \frac{4}{1}$$

$$y = -8 \cdot 4$$

$$y = -32$$

b.) $7 = \frac{2}{7}x$

$$x = \frac{7}{1} \cdot \frac{2}{7}$$

$$x = \frac{1}{1} \cdot \frac{2}{1} = 2$$

$$x = 2$$

c.) $\frac{1}{2}m = -\frac{3}{14}$

$$m = \frac{-3}{14} \cdot \frac{2}{1}$$

$$= \frac{-3}{7} \cdot \frac{1}{1}$$

$$= -\frac{3}{7} \cdot \frac{1}{1}$$

$$= -\frac{3}{7}$$

$$m = -\frac{3}{7}$$