7.5 Factoring Linear Expressions

When two or more numbers are multiplied, each number is a factor of the product.

Example 1: Name the factors of 24.

1, 2, 3, 4, 6, 8, 12, 24

Numbers that have only two factors, 1 and itself, are called prime numbers. They are whole numbers that are greater than 1.

Example 2: Name the first ten prime numbers.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, ...

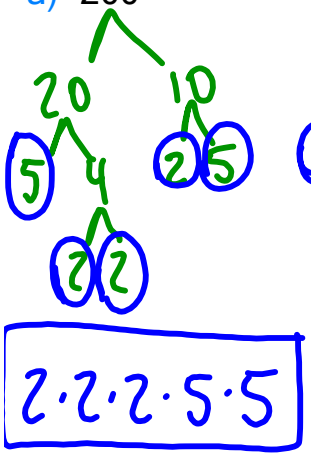
Whole numbers greater than 1 that are not prime are composite.

When a whole number is expressed as a product of factors that are all prime, the expression is called the prime factorization of the number.

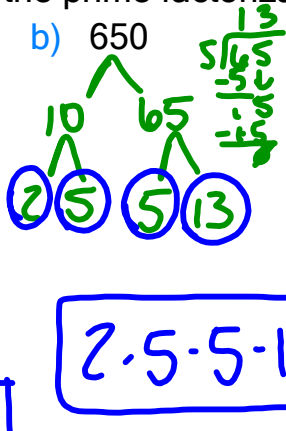
Example: The prime factorization of 18 is $2 \cdot 3 \cdot 3$.

Example 3: Find the prime factorization of the following numbers.

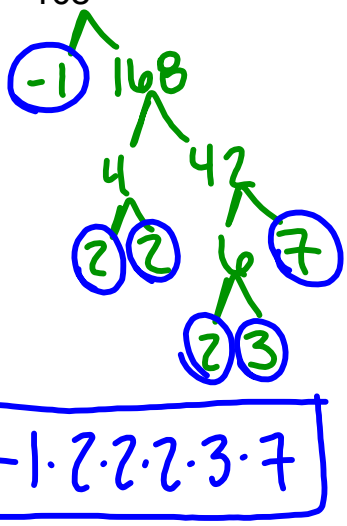
a) 200



b) 650



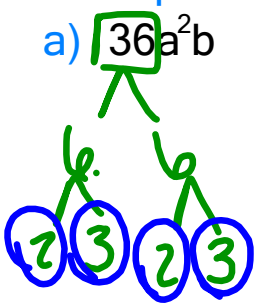
c) -168



A monomial is written in factored form when it is expressed asthe product of prime numbers and variables where no variable has an exponent greater than 1.

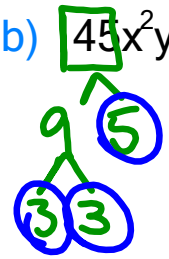
Example 4: Factor each monomial.

a) $36a^2b$



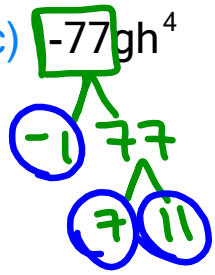
$2 \cdot 2 \cdot 3 \cdot 3 \cdot a \cdot a \cdot b$

b) $45x^2y^3$



$3 \cdot 3 \cdot 5 \cdot x \cdot x \cdot y \cdot y \cdot y$

c) $-77gh^4$

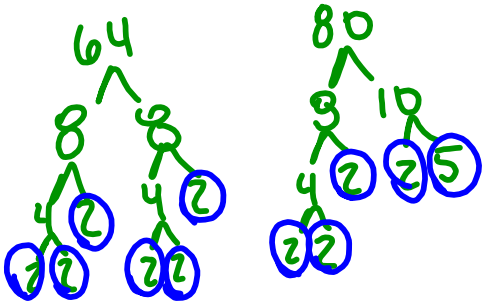


$-1 \cdot 7 \cdot 11 \cdot g \cdot h \cdot h \cdot h \cdot h$

The greatest common factor (GCF)of two or more monomials is the product of their common factors.

Example 5: Find the GCF of the following.

a) 64 and 80

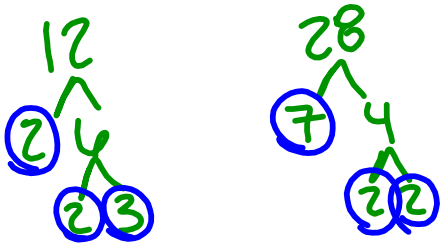


$64 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2$

$80 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 5$

$GCF = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

b) 12 and 28c



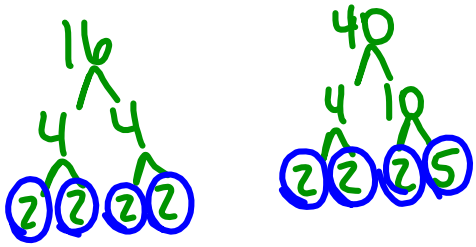
$12 = 2 \cdot 2 \cdot 3$

$28c = 2 \cdot 2 \cdot 7 \cdot c$

$GCF = 2 \cdot 2 = 4$

Example 7: Find the GCF of the following.

a) 16s and 40st

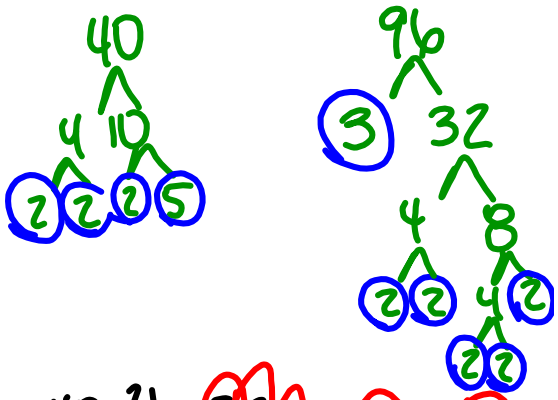


$16s = 2 \cdot 2 \cdot 2 \cdot 2 \cdot s$

$40st = 2 \cdot 2 \cdot 2 \cdot 5 \cdot s \cdot t$

$GCF = 2 \cdot 2 \cdot 2 \cdot s = \boxed{8s}$

b) $40a^2b$ and $96ab^3$



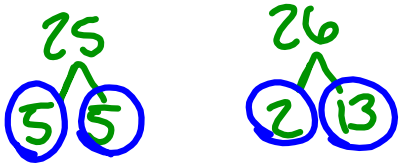
$40a^2b = 2 \cdot 2 \cdot 2 \cdot 5 \cdot a \cdot a \cdot b$

$96ab^3 = 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot a \cdot b \cdot b \cdot b$

$GCF = 2 \cdot 2 \cdot 2 \cdot a \cdot b = \boxed{8ab}$

Example 6: Find the GCF of the following.

a) 25x and 26y

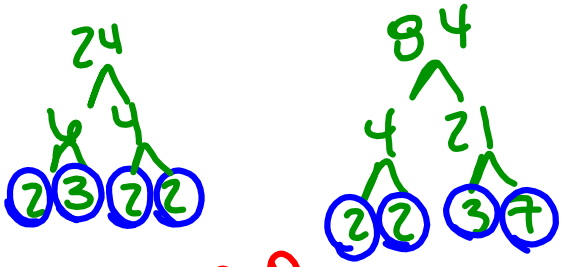


$25x = 5 \cdot 5 \cdot x$

$26y = 2 \cdot 13 \cdot y$

$\boxed{GCF = 1}$

b) $24x^2y^3z$ and $84xy^2z^3$



$24x^2y^3z = 2 \cdot 2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot y \cdot y \cdot y \cdot z$

$84xy^2z^3 = 2 \cdot 2 \cdot 3 \cdot 7 \cdot x \cdot y \cdot y \cdot z \cdot z \cdot z$

$GCF = 2 \cdot 2 \cdot 3 \cdot x \cdot y \cdot y \cdot z$

$= \boxed{12xy^2z}$