7.4 INVERSES FUNCTIONS

The inverse of a relation consisting of the ordered pairs (x,y) is the set of all ordered pairs (y,x).

The domain of the inverse is the range of the original relation.

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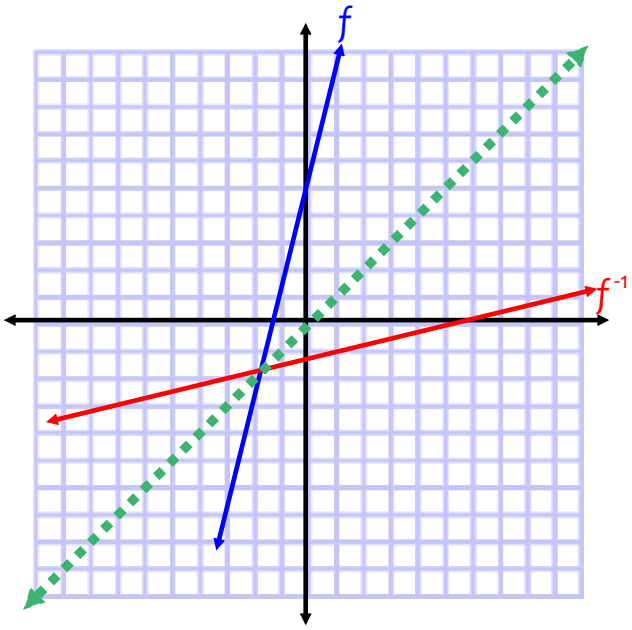
EXAMPLES: Find the inverse of each relation. State whether the relation is a function. State whether the inverse is a function.

1. {(1, 2), (2, 4), (3, 6), (4, 8)}

2. {(1, 5), (1, 6), (3, 6), (4, 9)}

If a function f and its inverse are **both functions**, the inverse of f is denoted by f^{-1}.



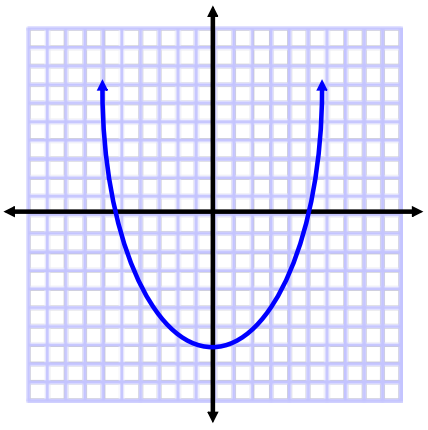
To find the inverse of a function, simply interchange x and y, and then solve for y.

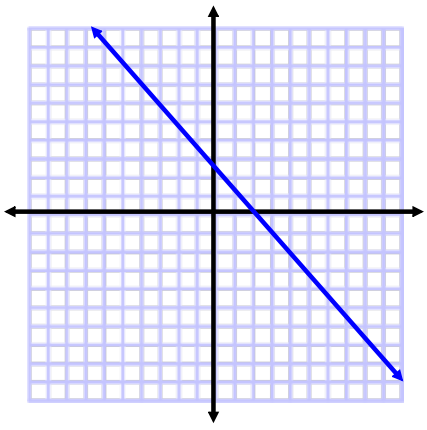
3. Find an equation for the inverse of $y = 3x - 2$.

4. Find an equation for the inverse of $y = 4x + 5$.

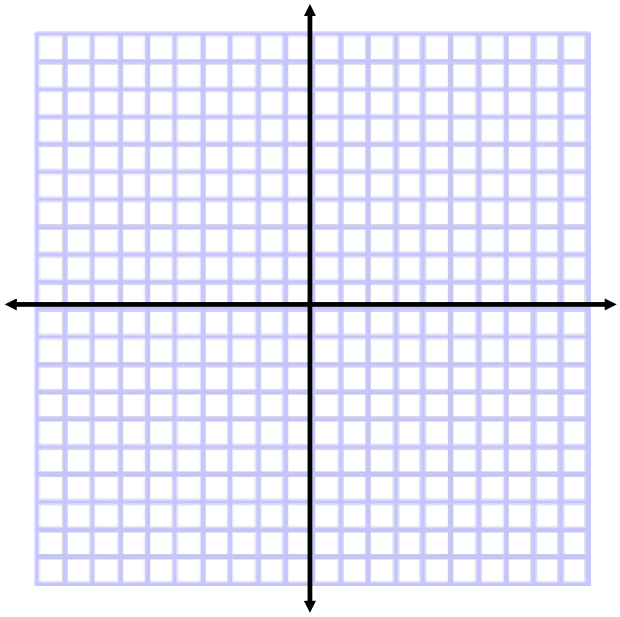
HORIZONTAL LINE TEST

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function at no more than one point.

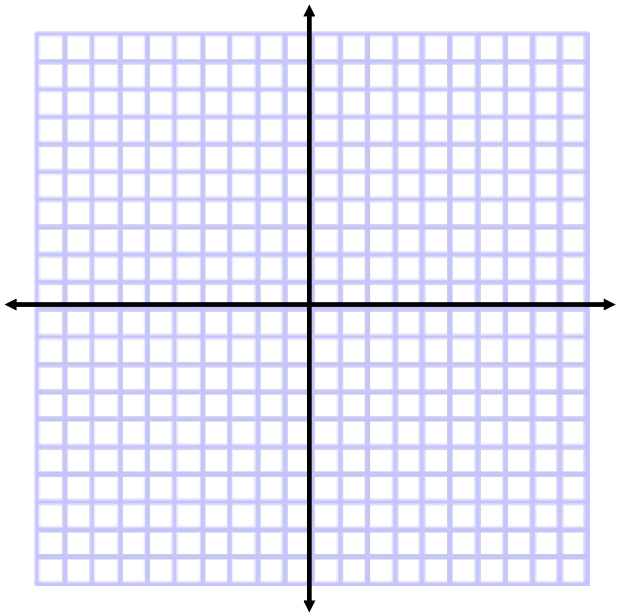




5. a) Graph the function $f(x) = 2x^2$.
   b) Then use the graph to determine whether the inverse is a function.
   c) Graph the inverse function.



6. a) Graph the function $f(x) = 2x + 5$.
   b) Then use the graph to determine whether the inverse is a function.
   c) Graph the inverse function.



COMPOSITION AND INVERSES

If f and g are functions and

$f(g(x)) = x$ and $g(f(x)) = x$,

then f and g are inverses of one another.

7. Show that $f(x) = 4x - 3$ and $g(x) = \frac{1}{4}x + \frac{3}{4}$ are inverses of each other.

$f(g(x)) = f\left(\frac{1}{4}x + \frac{3}{4}\right) = 4\left(\frac{1}{4}x + \frac{3}{4}\right) - 3 = 1x + 3 - 3 = x$

$\frac{4}{1}\cdot\frac{1}{4}$ $\quad$ $\frac{4}{1}\cdot\frac{3}{4}$

$g(f(x)) = g(4x-3) = \frac{1}{4}(4x-3) + \frac{3}{4} = 1x - \frac{3}{4} + \frac{3}{4} = x$

$\frac{1}{4}\cdot\frac{4}{1}$ $\quad$ $\frac{1}{4}\cdot\frac{3}{1}$

INVERSES

8. Show that $f(x) = 4x^2 - 8$ and $g(x) = \frac{\sqrt{x+8}}{2}$ are inverses of each other.

$f(g(x)) = f\left(\frac{\sqrt{x+8}}{2}\right) = 4\left(\frac{\sqrt{x+8}}{2}\right)^2 - 8 = 4\left(\frac{x+8}{4}\right) - 8$

$= x + 8 - 8 = x$

$g(f(x)) = g\left(4x^2 - 8\right) = \frac{\sqrt{4x^2 - 8 + 8}}{2} = \frac{\sqrt{4x^2}}{2} = \frac{\sqrt{4}\cdot\sqrt{x^2}}{2}$ $\quad$ $x^{\frac{2}{2}}$

$= \frac{2\cdot x^1}{2} = x$

INVERSES

9. Find the inverse function of $f(x) = 4x^3 + 1$.

$y = 4x^3 + 1$

$x = 4y^3 + 1$

$-1 \quad -1$

$\frac{x-1}{4} = \frac{4y^3}{4}$

$\sqrt[3]{\frac{x-1}{4}} = \sqrt[3]{y^3}$

$y = \sqrt[3]{\frac{x-1}{4}}$

10. Find the inverse function of $f(x) = -3x^7 - 5$.

$y = -3x^7 - 5$

$x = -3y^7 - 5$

$+5 \quad +5$

$\frac{x+5}{-3} = \frac{-3y^7}{-3}$

$\sqrt[7]{\frac{x+5}{-3}} = \sqrt[7]{y^7}$

$y = \sqrt[7]{\frac{x+5}{-3}}$