

7.4 INVERSES FUNCTIONS

The inverse of a relation consisting of the ordered pairs (x,y) is the set of all ordered pairs (y,x) .

The domain of the inverse is the range of the original relation.

The range of the inverse is the domain of the original relation.

EXAMPLES: Find the inverse of each relation.

State whether the relation is a function. State whether the inverse is a function.

x 's don't repeat in a function

1. $\{(1, 2), (2, 4), (3, 6), (4, 8)\}$
relation is a function
 ↑ ↑ ↑ ↑

2. $\{(1, 5), (1, 6), (3, 6), (4, 9)\}$
relation is not a function
 ↑ ↑ ↑ ↑

inverse: $\{(2, 1), (4, 2), (6, 3), (8, 4)\}$
 ↑ ↑ ↑ ↑

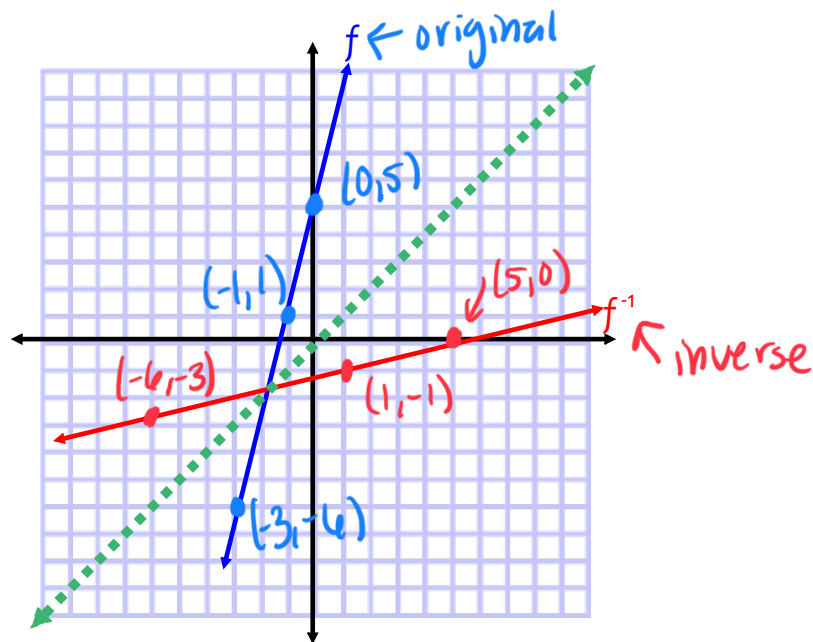
inverse is a function

inverse: $\{(5, 1), (6, 1), (6, 3), (9, 4)\}$
 ↑ ↑ ↑ ↑

inverse is not a function

7.4 Inverse Functions

If a function f and its inverse are **both functions**, the inverse of f is denoted by f^{-1} .



To find the inverse of a function, simply interchange x and y , and then solve for y .

3. Find an equation for the inverse of $y = 3x - 2$.

$$x = 3y - 2$$

$$\frac{x+2}{3} = \frac{3y}{3}$$

$$\boxed{y = \frac{x+2}{3}} \leftarrow \text{inverse}$$

7.4 Inverse Functions

4. Find an equation for the inverse of $y = 4x + 5$.

$$x = 4y + 5$$

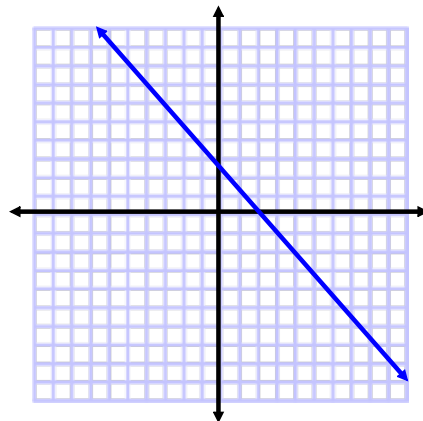
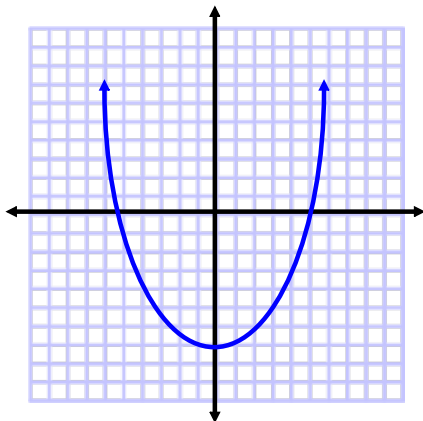
$$\frac{x-5}{4} = \frac{4y}{4}$$

$$y = \frac{x-5}{4} \quad \leftarrow \text{inverse}$$

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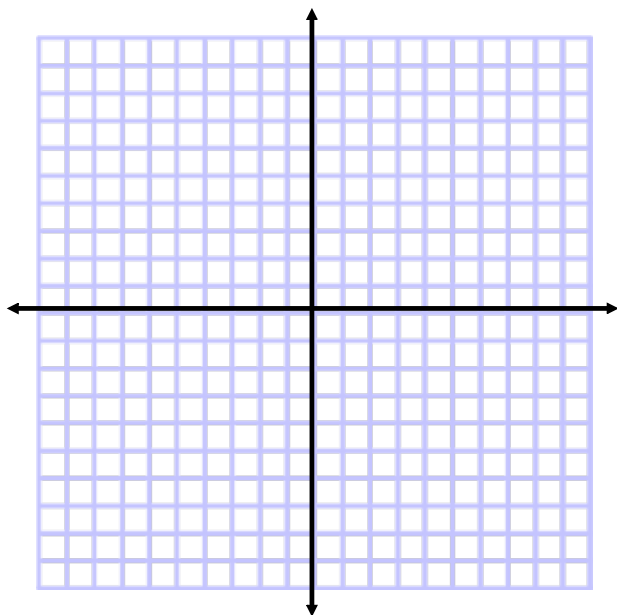
HORIZONTAL LINE TEST

The inverse of a function is a function if and only if every horizontal line intersects the graph of the given function at no more than one point.

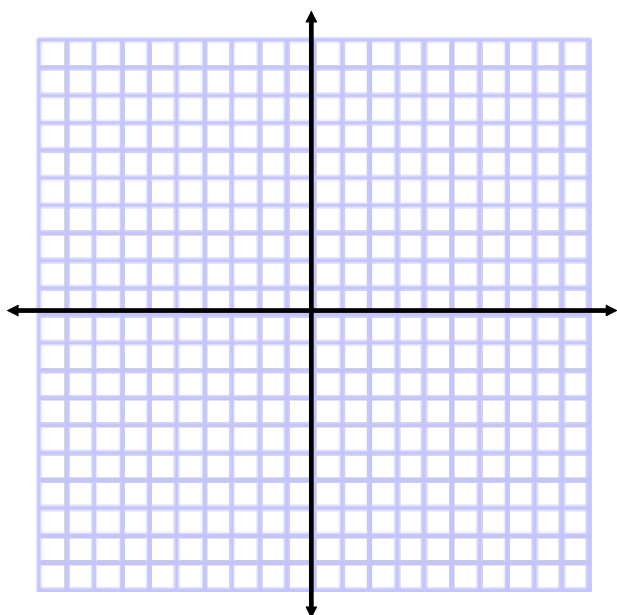


7.4 Inverse Functions

5. a) Graph the function $f(x) = 2x^2$.
- b) Then use the graph to determine whether the inverse is a function.
- c) Graph the inverse function.



6. a) Graph the function $f(x) = 2x + 5$.
- b) Then use the graph to determine whether the inverse is a function.
- c) Graph the inverse function.



COMPOSITION AND INVERSES

If f and g are functions and
 $f(g(x)) = x$ and $g(f(x)) = x$,
then f and g are inverses of one another.

7. Show that $f(x) = 4x - 3$ and $g(x) = \frac{1}{4}x + \frac{3}{4}$
are inverses of each other.

8. Show that $f(x) = 4x^2 - 8$ and $g(x) = \frac{\sqrt{x+8}}{2}$
are inverses of each other.

7.4 Inverse Functions

9. Find the inverse function of $f(x) = 4x^3 + 1$.

10. Find the inverse function of $f(x) = -3x^7 - 5$.