

6.4 Part 1 FACTORING POLYNOMIALS

I. Sum and Difference of Two Cubes:

- a) is a binomial,
- b) each term is perfect cube, and
- c) terms are connected by addition or subtraction.

Perfect Cubes:

| | | | |
|-------------|---------------|--------------------|------------------------|
| $1^3 = 1$ | $6^3 = 216$ | $(x)^3 = x^3$ | $(2x^2)^3 = 8x^6$ |
| $2^3 = 8$ | $7^3 = 343$ | $(x^2)^3 = x^6$ | $(6x^5)^3 = 216x^{15}$ |
| $3^3 = 27$ | $8^3 = 512$ | $(x^3)^3 = x^9$ | $(4x^3)^3 = 64x^9$ |
| $4^3 = 64$ | $9^3 = 729$ | $(x^4)^3 = x^{12}$ | |
| $5^3 = 125$ | $10^3 = 1000$ | $(x^5)^3 = x^{15}$ | |

FACTOR PATTERNS

SUM OF 2 CUBES: $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
 $(2d)^2 = 4d^2$

DIFFERENCE OF 2 CUBES: $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

Examples:

1. $x^3 - 1000$
 $(x)^3 \quad (10)^3$

$$(x - 10)(x^2 + 10x + 100)$$

2. $8d^3 + 1$
 $(2d)^3 \quad (1)^3$

$$(2d + 1)(4d^2 - 2d + 1)$$

Examples:

$$3. \quad 64y^3 - 1$$

$(4y)^3 \quad (1)^3$

$$(4y - 1)(16y^2 + 4y + 1)$$

$$4. \quad 512 + 27k^3$$

$(8)^3 \quad (3k)^3$

$$(8 + 3k)(64 - 24k + 9k^2)$$

$$5. \quad 216m^9 + 125$$

$$6. \quad 64 - 343h^3$$

II. GCF Revisited

REMEMBER!! The first step to factoring is LOOK FOR A GCF and factor out!

$$7. \quad 3d^3 - 81 \quad \text{GCF} = 3$$

$$3(d^3 - 27)$$

$(d)^3 \quad (3)^3$

$$3(d - 3)(d^2 + 3d + 9)$$

$$9. \quad 6w^4 + 48w$$

$$8. \quad 54p^3 + 2 \quad \text{GCF} = 2$$

$$2(27p^3 + 1)$$

$(3p)^3 \quad (1)^3$

$$2(3p + 1)(9p^2 - 3p + 1)$$

$$10. \quad 16x^5 - 250x^2$$

III. Factoring by Grouping

$$11. (x^3 - 3x^2) + 5x - 15$$

$$x^2(x-3) + 5(x-3)$$

$$(x-3)(x^2+5)$$

$$12. (f^3 - f^2) - 9f + 9$$

$$f^2(f-1) - 9(f-1)$$

$$(f-1)(f^2-9)$$

$$(f-1)(f-3)(f+3)$$

$$13. 27q^4 - 27q^3 + 8q - 8$$

$$14. 5t^4 + 5t^3 - 20t^2 - 20t$$

IV. Quadratic Techniques

A. Trinomials $ax^n + bx^{\frac{n}{2}} + c$

(exponent of middle term is half the exponent on leading term)

$$15. 4x^4 - 17x^2 + 4$$

$$16. 2h^4 - 9h^2 + 4$$

| | |
|----------|------------|
| Sum -17 | product 16 |
| -1 + -16 | -1 · -16 |

$$\frac{-1}{4} \quad \frac{-16}{1} = \frac{-4}{1}$$

$$(4x^2 - 1)(1x^2 - 4)$$

$$(2x-1)(2x+1)(x-2)(x+2)$$

17. $n^4 + 6n^2 + 5$

18. $x^4 - 6x^2 - 27$

B. Binomials (Difference of Squares) $(a^2 - b^2) = (a - b)(a + b)$

19. $16j^4 - 25$
 $(4j^2)^2 \quad (5)^2$

$$(4j^2 - 5)(4j^2 + 5)$$

21. $64x^6 - 1$

20. $2z^5 - 32z$
 $2z(z^4 - 16)$

$$2z(z^2 - 4)(z^2 + 4)$$

$$2z(z - 2)(z + 2)(z^2 + 4)$$

$$2z(z - 2)(z + 2)(z^2 + 4)$$

