# 5.4 Operations with Complex Numbers

## Square Root of Negative Numbers

The square root of a negative real number has TWO imaginary roots: one positive, one negative.

$$\sqrt{-r} = \sqrt{-1} \cdot \sqrt{r} \left( \text{where } \boxed{\sqrt{-1} = i} \right) = i\sqrt{r}$$

and

$$\boxed{i^2} = i \cdot i = \sqrt{-1} \cdot \sqrt{-1} = \boxed{-1}$$

Examples: Simplify.

1. $\sqrt{-81} = 9i$

2. $\sqrt{-48}$ $4\sqrt{3}\,i$

   $2\,|\,48$, $2\,|\,24$, $2\,|\,12$, $2\,|\,6$, $3$

   $\boxed{4i\sqrt{3}}$

3. $\sqrt{-120}$ $2\sqrt{30}\,i$

   $\boxed{2i\sqrt{30}}$

   $2\,|\,120$, $2\,|\,60$, $2\,|\,30$, $3\,|\,15$, $5$

4. $\sqrt{-256}$

   $\boxed{16i}$

   $2\,|\,256$, $2\,|\,128$, $2\,|\,64$, $2\,|\,32$, $2\,|\,16$, $2\,|\,8$, $2\,|\,4$, $2$

Examples: Solve by taking square roots.

5. $x^2 + 16 = 0$

$-16 \quad -16$

$\sqrt{x^2} = \sqrt{-16}$

$\boxed{x = \pm 4i}$

7. $-3x^2 - 10 = 44$

$+10 \quad +10$

$\frac{-3x^2}{-3} = \frac{54}{-3}$

$\sqrt{x^2} = \sqrt{-18}$

$x = \frac{\pm}{3\sqrt{2}} i$

$\boxed{x = \pm 3i\sqrt{2}}$

2 | 18
3 | 9
3

6. $2x^2 + 68 = 20$

$-68 \quad -68$

$\frac{2x^2}{2} = \frac{-48}{2}$

$\sqrt{x^2} = \sqrt{-24}$

2 | 24
2 | 12
2 | 6
3

$x = \pm 2\sqrt{6}\, i$

$\boxed{x = \pm 2i\sqrt{6}}$

8. $\frac{1}{4}x^2 + 10 = -15$

$-10 \quad -10$

$4 \cdot \frac{1}{4}x^2 = -25 \cdot 4$

$\sqrt{x^2} = \sqrt{-100}$

$\boxed{x = \pm 10i}$

Examples: Solve by taking square roots.

9. $2(x - 1)^2 + 12 = 0$

$-12 \quad -12$

$\frac{2(x-1)^2}{2} = \frac{-12}{2}$

$\sqrt{(x-1)^2} = \sqrt{-6}$

$x - 1 = \pm i\sqrt{6}$

$+1 \quad +1$

$\boxed{x = 1 \pm i\sqrt{6}}$

11. $-5(x + 2)^2 - 7 = 38$

12. $-\frac{1}{3}(x - 7)^2 + 5 = 23$

10. $\frac{1}{2}(x + 4)^2 - 8 = -26$

$+8 \quad +8$

$2 \cdot \frac{1}{2}(x+4)^2 = -18 \cdot 2$

$\sqrt{(x+4)^2} = \sqrt{-36}$

$x + 4 = \pm 6i$

$-4 \quad -4$

$\boxed{x = -4 \pm 6i}$

The standard form of a complex number is

$a + bi$

real part — imaginary part

Every number can be written as a <u>complex number</u>.

$9 + 2i \longrightarrow$ imaginary number

$0 + 2i \longrightarrow$ pure imaginary number

$9 + 0i \longrightarrow$ real number

## Adding and Subtracting Complex Numbers

| | |
|---|---|
| Add or subtract: | real part to real part |
| | imaginary part to imaginary part |

<u>Examples</u>: Simplify.

13. $(4 + i) + (3 + 2i)$ $\boxed{7 + i}$

14. $(7 - 5i) - (1 - 5i)$

$7 - 5i - 1 + 5i$ $\quad 6 + 0i = \boxed{6}$

15. $6 - (-2 + 9i) + (-8 + 4i)$

$6 + 2 - 9i + -8 + 4i$ $\quad 0 - 5i = \boxed{-5i}$

16. $2i - (3 + i) + (2 - 3i)$

$2i - 3 - i + 2 - 3i$ $\quad \boxed{-1 - 2i}$

## Multiplying Complex Numbers

Examples: Simplify.

17. $5i(-2 + i)$

18. $(-1 + 2i)(7 - 4i)$

19. $(6 + 3i)(6 - 3i)$

20. $(2 + 5i)^2$

## Dividing Complex Numbers

A. Pure Imaginary Denominator

Multiply numerator and denominator by i.

Examples: Simplify.

21. $\frac{2 + 8i}{i}$

22. $\frac{3 + 7i}{2i}$

23. $\frac{4 - i}{2i}$

## B. Imaginary Denominator a + bi

To simplify a fraction with an imaginary number, a + bi, in the denominator, you must multiply by the conjugate of the denominator.

Example: The conjugate of $1 - 3i$ is $1 + 3i$.

The conjugate of $6 + 5i$ is $6 - 5i$.

Examples: Simplify.

24. $\frac{2}{7 - 8i}$

Examples: Simplify.

25. $\frac{-3}{2 + i}$

26. $\frac{3 + 11i}{-1 - 2i}$