5.3 Solve Quadratics by Square Roots

I. Square Root of Positive Number

A number r is a square root of s if $r^2 = s$

$\sqrt{s} = r$ iff $r^2 = s$

radical sign

radicand (number under the radical sign)

A positive number has TWO square roots:

$\sqrt{s}$ and $-\sqrt{s}$

$\sqrt{100} = 10$, since $10^2 = 100$

$\sqrt{100} = -10$, since $(-10)^2 = 100$

To simplify a radical (if you do not know the root/answer), factor the radicand using prime factors.

prime factors: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, etc.

EXAMPLES:

1. $\sqrt{196}$

2	196
2	98
7	49
	7

$2 \cdot 7$

14

2. $\sqrt{80}$

2	80
2	40
2	20
2	10
	5

$2 \cdot 2$

$\boxed{4\sqrt{5}}$

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KEY CONCEPT *For Your Notebook*

Properties of Square Roots ($a > 0, b > 0$)

Product Property	$\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$	**Example**	$\sqrt{18} = \sqrt{9} \cdot \sqrt{2} = 3\sqrt{2}$
Quotient Property	$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	**Example**	$\sqrt{\frac{2}{25}} = \frac{\sqrt{2}}{\sqrt{25}} = \frac{\sqrt{2}}{5}$

A square root is simplified when:
1) the radicand has NO perfect square factors other than 1,
2) the radicand is NOT a fraction, and
3) NO radical is in the denominator.

3. $\cancel{\sqrt{\ }}(x+5)^{\cancel{2}}$

$\boxed{x+5}$

6. $\cancel{\sqrt{\ }}(x-4)^{\cancel{2}}$

$\boxed{x-4}$

4. $\sqrt{3} \bullet \sqrt{75}$

$5 \mid 75$
$5 \mid 15$
3

$\sqrt{3} \cdot 5\sqrt{3}$

$5\sqrt{9} = 5 \cdot 3 = \boxed{15}$

7. $\sqrt{6} \bullet \sqrt{21} = \sqrt{126}$

$2 \mid 126$
$3 \mid 63$
$3 \mid 21$
7

$\boxed{3\sqrt{14}}$

5. $\sqrt{\frac{4}{81}} = \frac{\sqrt{4}}{\sqrt{81}} = \boxed{\frac{2}{9}}$

8. $\frac{\sqrt{7}}{\sqrt{16}} = \boxed{\frac{\sqrt{7}}{4}}$

II. Rationalize Denominator Containing Square Root

To rationalize transforms a fraction to an equivalent form with NO radical in the denominator.

Steps:

1. Reduce the fraction, if possible.
   Reduce like parts: radicand to radicand; coefficient to coefficient
2. Multiply top and bottom by the square root in the denominator.
3. Simplify top and bottom. Reduce again, if possible.

EXAMPLES:

9. $\frac{1}{\sqrt{2}}\left(\frac{\cdot\sqrt{2}}{\cdot\sqrt{2}}\right) = \frac{\sqrt{2}}{2}$

11. $\frac{\sqrt{12}}{\sqrt{18}} = \sqrt{\frac{12 \div 6}{18 \div 6}}$

$\sqrt{\frac{2}{3}} = \frac{\sqrt{2}\cdot\sqrt{3}}{\sqrt{3}\cdot\sqrt{3}} = \boxed{\frac{\sqrt{6}}{3}}$

10. $\frac{6}{\sqrt{3}}\frac{\cdot\sqrt{3}}{\cdot\sqrt{3}} = \frac{\overset{2}{\cancel{6}}\sqrt{3}}{\underset{1}{\cancel{3}}}$

$= \boxed{2\sqrt{3}}$

12. $\frac{\sqrt{64}}{\sqrt{16}} = \frac{8}{4} = \boxed{2}$

III. Solve Quadratic Equation By Square Roots

When solving a quadratic equation by square roots, you are finding both the positive and negative roots.

$$\boxed{\text{If } x^2 = a \text{ then } x = \pm\sqrt{a}.}$$

NOTE: To solve by square roots, you must isolate x^2 on one side of the equation.

A. Form $ax^2 = c$ or $ax^2 - c = 0$ (no bx term)

EXAMPLES:

13. $4x^2 = 240$

$$\frac{4x^2}{4} = \frac{240}{4}$$

$$\sqrt{x^2} = \sqrt{60}$$

$$x = \pm\sqrt{60}$$

$$\boxed{x = \pm 2\sqrt{15}}$$

$2 \mid 60$, $2 \mid 30$, $3 \mid 15$, 5

14. $2x^2 + 7 = 88$

$$-7 \quad -7$$

$$\frac{2x^2}{2} = \frac{81}{2}$$

$$\sqrt{x^2} = \sqrt{\frac{81}{2}}$$

$$x = \pm\frac{\sqrt{81}}{\sqrt{2}}$$

$$x = \pm\frac{9}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$

$$\boxed{x = \pm\frac{9\sqrt{2}}{2}}$$

B. Form $a(x-h)^2 + k = 0$ or $a(x-h)^2 = k$.

Still solving for x!!!

Isolate parentheses. Take square root of both sides.
Then add or subtract to isolate x.

EXAMPLES:

15. $(x+3)^2 - 25 = 0$

$+25 \quad +25$

$\sqrt{(x+3)^2} = \sqrt{25}$

$x+3 = \pm 5$

$-3 \quad -3$

$x = -3 \pm 5$

$-3+5 \qquad -3-5$

$2 \qquad -8$

$\boxed{x = -8, 2}$

16. $-2(x-1)^2 = -12$

$\overline{-2} \quad \overline{-2}$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x-1 = \pm\sqrt{6}$

$+1 \quad +1$

$\boxed{x = 1 \pm \sqrt{6}}$

17. $\frac{1}{3}(x-2)^2 - 4 = 11$

$+4 \quad +4$

$3 \cdot \frac{1}{3}(x-2)^2 = 15 \cdot 3$

$\sqrt{(x-2)^2} = \sqrt{45}$

$x-2 = \pm\sqrt{45}$

$+2 \quad +2$

$x = 2 \pm \sqrt{45}$

$5 \,|\, 45$

$3 \,|\, 9$

3

$\boxed{x = 2 \pm 3\sqrt{5}}$

18. $5(x+3)^2 + 9 = 20$

$-9 \quad -9$

$\frac{5(x+3)^2}{5} = \frac{11}{5}$

$\sqrt{(x+3)^2} = \sqrt{11/5}$

$x+3 = \pm\sqrt{\frac{11}{5}}$

$-3 \quad -3$

$x = -3 \pm \frac{\sqrt{11}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\boxed{x = -3 \pm \frac{\sqrt{55}}{5}}$

Attachments

PRACTICE WORKSHEET Square Roots and Quad Equations.doc

Worksheet Simplify Square Roots.doc