12.3 Properties of Logarithms

Product Property $\log_b m + \log_b n = \log_b mn$

Quotient Property $\log_b m - \log_b n = \log_b \frac{m}{n}$

Power Property $n \log_b m = \log_b m^n$

Properties of Logarithms

1. $\log_b 1 = 0$ $b^0 = 1$
2. $\log_b b = 1$ $b^1 = b$
3. $\log_b b^x = x$
4. $b^{\log_b x} = x$

One-to-One Property of Logarithms

If $\log_b x = \log_b y$, then $x = y$.

Examples

Write each expression as a single logarithm. Then simplify, if possible.

1. $\log_2 5 + \log_2 7$

$\log_2 5 \cdot 7 = \log_2 35$

2. $\log_3 45 - \log_3 9$

$\log_3 \frac{45}{9} = \log_3 5$

3. $\log_2 5 + \log_2 x - \log_2 10$

$\log_2 5x - \log_2 10$

$\log_2 \frac{5x}{10}$

$\log_2 \frac{x}{2}$

4. $\log_7 3x - \log_7 9x + \log_7 6y$

$\log_7 \frac{3x}{9x}$

$\log_7 \frac{1}{3} + \log_7 6y$

$\log_7 \frac{1}{3} \cdot 6y = \log_7 2y$

5. $5\log_2 m - 2\log_2 n$

$\log_2 m^5 - \log_2 n^2 = \log_2 \frac{m^5}{n^2}$

6. $4\log_b m + \log_b n - \frac{1}{2}\log_b p$

$\log_b m^4 + \log_b n - \log_b p^{1/2}$

$\log_b m^4 n - \log_b p^{1/2}$

$\log_b \frac{m^4 n}{p^{1/2}}$

Examples

Write each expression as a sum or difference of logarithms. Then simplify, if possible.

11. $\log_2 4x^3$

$\log_2 4 + \log_2 x^3$

$\log_2 4 + 3\log_2 x = \log_2 2^2 + 3\log_2 x$

$= 2 + 3\log_2 x$

12. $\log_4 \frac{4a}{b^2}$

$\log_4 4a - \log_4 b^2$

$\log_4 4^1 + \log_4 a - 2\log_4 b$

$1 + \log_4 a - 2\log_4 b$

13. $\log_3 \frac{k^3 m}{9}$

$\log_3 k^3 m - \log_3 9$

$\log_3 k^3 + \log_3 m - \log_3 9$

$3\log_3 k + \log_3 m - \log_3 3^2$

$3\log_3 k + \log_3 m - 2$