# 11.2 Part 2 Arithmetic Series

Series -> the indicated sum of the terms of a sequence

The symbol $S_n$ is used to represent the sum of the first n terms of a series.

We use a special formula to find the arithmetic series without having to add all of the terms together.

## Sum of An Arithmetic Series

The sum, $S_n$, of the first n terms of an arithmetic series is given by the following formula:

$$S_n = \frac{n}{2}(a_1 + a_n)$$

This formula is used when the first term, the $n^{th}$ term, and the number of terms are known.

Using the formula from the previous section, we can substitute that $a_n$ into the series equation.

$$S_n = \frac{n}{2}[2a_1 + (n - 1)d]$$

We can use this formula when we know the first term, the number of terms, and the common difference.

Example: Find $S_n$ for each arithmetic series described.

$a_1 = 2, \quad a_n = 200, \quad n = 100$

$$\begin{array}{r} 202 \\ \times\ \ 5 \\ \hline 10100 \end{array}$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$$= \frac{100}{2}(2 + 200)$$

$$= 50(202)$$

$$\boxed{S_n = 10,100}$$

Example: Find $S_n$ for each arithmetic series described.

$a_1 = 5, \quad a_n = 100, \quad n = 200$

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$= \frac{200}{2}(5 + 100)$$
$$= 100(105)$$
$$\boxed{S_n = 10500}$$

Example: Find $S_n$ for each arithmetic series described.

$a_1 = 4, \quad n = 15, \quad d = 3$

$$S_n = \frac{n}{2}[2a_1 + (n-1)d]$$
$$= \frac{15}{2}[2(4) + (15-1)(3)]$$
$$= \frac{15}{2}[8 + (14)(3)] = \frac{15}{2}[8 + 42]$$
$$= \frac{15}{\cancel{2}}[\cancelto{25}{50}] = 15 \cdot 25$$
$$\boxed{S_n = 375}$$

$$\begin{array}{r} \overset{2}{2}5 \\ \times 15 \\ \hline 125 \\ 250 \\ \hline 375 \end{array}$$

Example: Find $S_n$ for each arithmetic series described.

$a_1 = 50, \quad n = 20, \quad d = -4$

$$S_n = \frac{n}{2}\left[2a_1 + (n-1)d\right]$$
$$= \frac{20}{2}\left[2 \cdot 50 + (20-1)(-4)\right]$$
$$= 10\left[100 + (19)(-4)\right]$$
$$= 10\left[100 + -76\right] = 10[24]$$
$$\boxed{S_n = 240}$$

Example: Find $S_n$ for each arithmetic series described.

$a_1$ $\qquad a_n = -39$

$-3 + (-7) + (-11) + \ldots + (-39)$ $\quad n = ?$

$d = -4$

$$a_n = a_1 + (n-1)d$$
$$-39 = -3 + (n-1)(-4)$$
$$-39 = -3 + -4n + 4$$
$$-39 = 1 + -4n$$
$$-1 \quad -1$$
$$\frac{-40}{-4} = \frac{-4n}{-4}$$
$$n = 10$$

$$S_n = \frac{n}{2}(a_1 + a_n)$$
$$= \frac{10}{2}(-3 + -39)$$
$$= 5(-42)$$
$$\boxed{S_n = -210}$$

Example: Find $S_n$ for each arithmetic series described.

$a_1$ $\qquad$ $a_n$

$9 + 11 + 13 + 15 + \ldots + 31$ $\qquad n = ?$

$d=2$

$a_n = a_1 + (n-1)d$

$31 = 9 + (n-1)(2)$

$31 = 9 + 2n - 2$

$31 = 7 + 2n$

$-7 \quad -7$

$\frac{24}{2} = \frac{2n}{2}$ $\qquad n = 12$

$S_n = \frac{n}{2}(a_1 + a_n)$

$= \frac{12}{2}(9+31)$

$= 6(40)$

$\boxed{S_n = 240}$

A sigma notation can also be used to express an arithmetic series.

The upper number is the upper limit of the index. To generate the terms of the series, successively replace the index of summation with each value of the variable.

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1 = 3(1) + 7$
$= 3 + 7 = 10$

Example: Find the sum.

$= \frac{6}{2}(10 + 25)$
$= 3(35)$

$a_n = 3(6) + 7$
$= 18 + 7 = 25$

$$\sum_{n=1}^{6} (3n + 7)$$

$\boxed{S_n = 105}$

1, 2, 3, 4, 5, 6 $n = 6$

$$S_n = \frac{n}{2}(a_1 + a_n)$$

$a_1 = 3(5) - 6$
$= 15 - 6$
$= 9$

Example: Find the sum.

$= \frac{7}{2}(9 + 27)$
$= \frac{7}{\cancel{2}}(\cancel{36}\,18)$

$a_n = 3(11) - 6$
$= 33 - 6$
$= 27$

$$\sum_{i=5}^{11} (3i - 6)$$

$\boxed{S_n = 126}$

$n = 7$
5, 6, 7, 8, 9, 10, 11