11.4 Infinite Geometric Series

Sum of an Infinite Geometric Series

The sum, S, of a geometric series where

$-1 < r < 1$ is given by the following formula:

$$S = \frac{a_1}{1 - r}$$

If $|r| > 1$, then the infinite geometric series does not have a sum. Therefore, we write "does not exist",

Example: Find the sum of each infinite geometric series.

$a_1 = 6 \qquad r = \frac{1}{3}$

$$S = \frac{a_1}{1-r} = \frac{6}{\frac{3}{3}\cancel{1} - \frac{1}{3}} = \frac{6}{2/3} = \frac{\overset{3}{\cancel{6}}}{1} \cdot \frac{3}{\underset{1}{\cancel{2}}}$$

$$= \boxed{9}$$

<u>Example:</u> Find the sum of each infinite geometric series.

$a_1 = -4 \qquad r = \frac{-1}{2}$

$$S = \frac{a_1}{1-r} = \frac{-4}{\frac{2}{2}\cancel{1} + \frac{1}{2}} = \frac{-4}{3/2} = \frac{-4}{1} \cdot \frac{2}{3} = \boxed{\frac{-8}{3}}$$

<u>Example:</u> Find the sum of each infinite geometric series.

$r = \frac{4}{8} = \frac{1}{2}$

$8 + 4 + 2 + \ldots$

$$S = \frac{a_1}{1-r} = \frac{8}{\frac{2}{2}\cancel{1} - \frac{1}{2}} = \frac{8}{1/2} = \frac{8}{1} \cdot \frac{2}{1} = \boxed{16}$$

<u>Example:</u> Find the sum of each infinite geometric series.

$r = \frac{-1/3}{1} = -\frac{1}{3}$

$$1 - \frac{1}{3} + \frac{1}{9} - \ldots$$

$S = \frac{a_1}{1-r} = \frac{1}{\frac{3}{3}\cancel{1} + \frac{1}{3}} = \frac{1}{4/3} = \boxed{\frac{3}{4}}$

<u>Example:</u> Find the sum of each infinite geometric series.

$r = \frac{3/2}{1} = \frac{3}{2} = 1\frac{1}{2}$

$$1 + \frac{3}{2} + \frac{9}{4} + \ldots$$

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Example: Find the first three terms of each infinite geometric series:

$S = 4 \qquad r = \frac{1}{3}$

$$(1-r)\cdot S = \frac{\boxed{a_1}}{1-r}\cdot(1-r)$$

$$a_1 = S(1-r)$$

$$= (4)\left(\cancel{1}^{3/3} - \frac{1}{3}\right)$$

$$= \frac{4}{1}\cdot\left(\frac{2}{3}\right) = \frac{8}{3}$$

$$a_1 = \boxed{\frac{8}{3}}$$

$$a_2 = \frac{8}{3}\cdot\frac{1}{3} = \boxed{\frac{8}{9}}$$

$$a_3 = \frac{8}{9}\cdot\frac{1}{3} = \boxed{\frac{8}{27}}$$

Example: Find the first three terms of each infinite geometric series:

$S = 64 \qquad r = \frac{-3}{4}$

$$a_1 = S(1-r) = 64\left(\cancel{1}^{4/4} + \frac{3}{4}\right)$$

$$= \cancel{64}^{16}\left(\frac{7}{\cancel{4}}\right) = 112$$

$$a_1 = \boxed{112}$$

$$a_2 = \frac{\cancel{112}^{28}}{1}\cdot\frac{-3}{\cancel{4}} = \boxed{-84}$$

$$a_3 = -\cancel{84}^{21}\cdot\frac{-3}{\cancel{4}} = \boxed{63}$$