11.3 Part 2 Geometric Series (of a Finite Number of Terms)

The sum of the terms of a geometric sequence is called a GEOMETRIC SERIES.

Sum of a Geometric Series Formula:

$$S_n = \frac{a_1 - a_1r^n}{1 - r} \quad \text{or} \quad S_n = \frac{a_1(1 - r^n)}{1 - r}$$

where $r \neq 1$

Find the sum of the $\boxed{\text{first five}}$ terms of a geometric series for which $a_1 = 4$ and $r = -3$.

$n=5$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{4(1-(-3)^5)}{1+3}$$

$$= \frac{4(1+243)}{4} = \frac{\cancel{4}^{1}(244)}{\cancel{4}_{1}} = \boxed{244}$$

Find the sum of each geometric series:

$a_1 = 16 \qquad r = \frac{1}{2} \qquad n = 7$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{16\left(1-\left(\frac{1}{2}\right)^7\right)}{\cancel{1}^{\frac{2}{2}}-\frac{1}{2}}$$

$$= \frac{16\left(\cancel{1}^{\frac{128}{128}}-\frac{1}{128}\right)}{1/2} = \frac{\frac{16}{1}\left(\frac{127}{128}\right)}{1/2} = \frac{\frac{2032}{128}}{1/2}$$

$$= \frac{2032}{\cancel{128}_{64}} \cdot \frac{\cancel{2}^{1}}{1} = \frac{2032}{64} = \boxed{\frac{127}{4} \text{ OR } 31.75}$$

Find the sum of each geometric series:

$a_1 = 4 \qquad r = \frac{-1}{2} \qquad n = 8$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{4\left(1-\left(\frac{-1}{2}\right)^8\right)}{\cancel{1}^{\frac{2}{2}}-\frac{-1}{2}} = \frac{4\left(\cancel{1}^{\frac{256}{256}}-\frac{1}{256}\right)}{3/2}$$

$$= \frac{\cancel{4}^{1}\left(\frac{255}{\cancel{256}_{64}}\right)}{3/2} = \frac{255/64}{3/2} = \frac{\cancel{255}^{85}}{\cancel{64}_{32}} \cdot \frac{\cancel{2}^{1}}{\cancel{3}_{1}}$$

$$= \boxed{\frac{85}{32} \text{ or } 2.65625}$$

For when you know the n^{th} term, the first term, and the common ratio, you can use this formula to solve for the geometric series:

$$S_n = \frac{a_1 - a_n r}{1 - r}$$

Find the sum of a geometric series for which $a_1 = 4$, $a_n = 256$, and $r = 4$.

$$S_n = \frac{a_1 - a_n r}{1 - r} = \frac{4 - 256(4)}{1 - 4}$$

$$= \frac{4 - 1024}{-3} = \frac{-1020}{-3} = \boxed{340}$$

Find a_1 in a geometric series where

$S_7 = 3{,}279$ and $r = 3$.

$$(1-r)S_n = \frac{\boxed{a_1}(1-r^n)}{1-r}\cdot(1-r)$$

$$\frac{S_n(1-r)}{(1-r^n)} = \frac{\boxed{a_1}(1-r^n)}{(1-r^n)}$$

$$a_1 = \frac{S_n(1-r)}{1-r^n}$$

$$a_1 = \frac{S_n(1-r)}{1-r^n}$$

$$= \frac{3279(1-3)}{1-3^7}$$

$$= \frac{3279(-2)}{1-2187}$$

$$= \frac{-6558}{-2186}$$

$$\boxed{a_1 = 3}$$

Find a_1 in a geometric series where

$S_8 = 13{,}120$ and $r = 3$.

$$a_1 = \frac{S_n(1-r)}{1-r^n} = \frac{13120(1-3)}{1-3^8}$$

$$= \frac{13120(-2)}{1-6561} = \frac{-26240}{-6560}$$

$$\boxed{a_1 = 4}$$

Find the sum:

Sequence: $a_n = a_1 r^{n-1}$

$n=5$, a_1, r

$$\sum_{j=1}^{5} 2(4)^{j-1}$$

$$S_n = \frac{a_1(1-r^n)}{1-r} = \frac{2(1-(4)^5)}{1-4} = \frac{2(1-1024)}{-3}$$

$$= \frac{2(-1023)}{-3} = \frac{-2046}{-3} = \boxed{682}$$

Use sigma notation to express:

Sequence: $a_n = a_1 r^{n-1}$

a_1

$\boxed{1} - 3 + 9 - 27 + 81 - 243$

$r = \frac{-3}{1} = -3$

$$\boxed{\sum_{x=1}^{6} (1)(-3)^{x-1}}$$