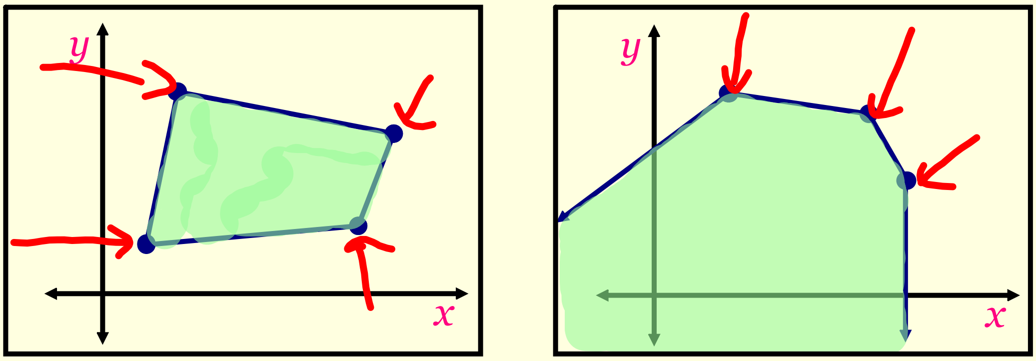
3.4 LINEAR PROGRAMMING & OPTIMIZATION

- Used to find optimal solutions
- Include the following characteristics:
  - The inequalities contained in the problem are called constraints.
  - The solution to the set of constraints is called the feasible region.
  - The function to be maximized or minimized is called the objective function.

CORNER-POINT PRINCIPLE

In linear programming, the maximum and minimum values of the objective function each occur at one of the vertices of the feasible region.



Bounded Region

Unbounded Region

EXAMPLE 1

objective function

$P = 3x + y$

A. Graph the feasible region.

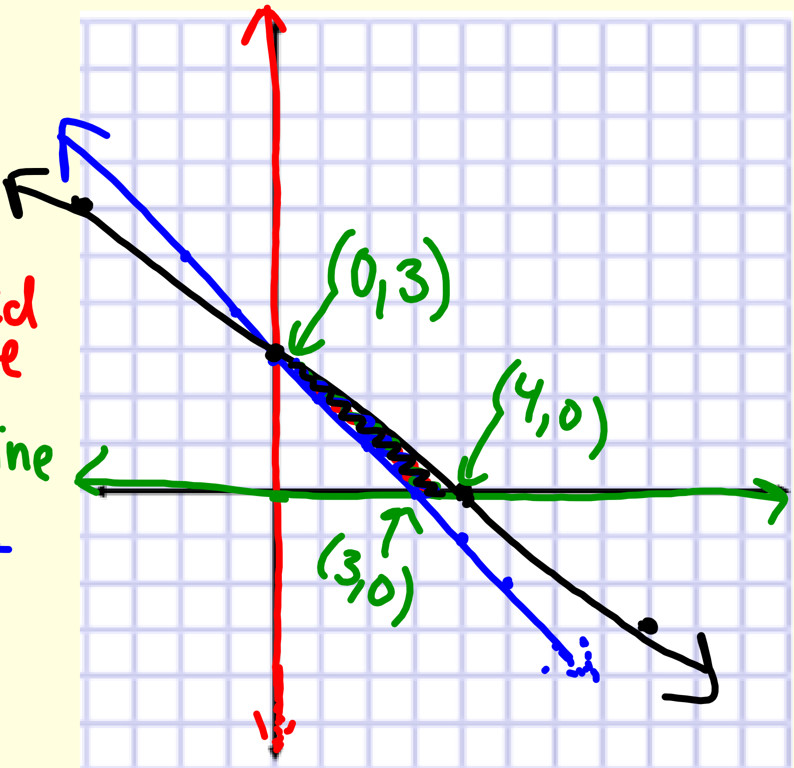
B. Find the maximum and minimum values, if they exist.

Constraints:

$$\begin{cases} x + y \ge 3 \\ 3x + 4y \le 12 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

$x \ge 0$ vertical solid line

$y \ge 0$ horizontal solid line



$x + y \ge 3$
$-x \qquad -x$
$y \ge 3 - x$ $\quad m = \frac{-1}{1}$ $\quad b = 3$

$3x + 4y \le 12$
$-3x \qquad -3x$

$\frac{4y}{4} \le \frac{12}{4} - \frac{3x}{4}$ $\quad m = \frac{-3}{4}$

$y \le 3 - \frac{3}{4}x$ $\quad b = 3$

$P = 3x + y$

x y (0,3)	(3,0)	(4,0)
$P = 3(0) + 3$	$P = 3(3) + 0$	$P = 3(4) + 0$
$= 0 + 3$	$= 9 + 0$	$= 12 + 0$
$\boxed{P = 3}$ min	$P = 9$	$\boxed{P = 12}$ max

EXAMPLE 2

objective function

$E = x + y$

A. Graph the feasible region.

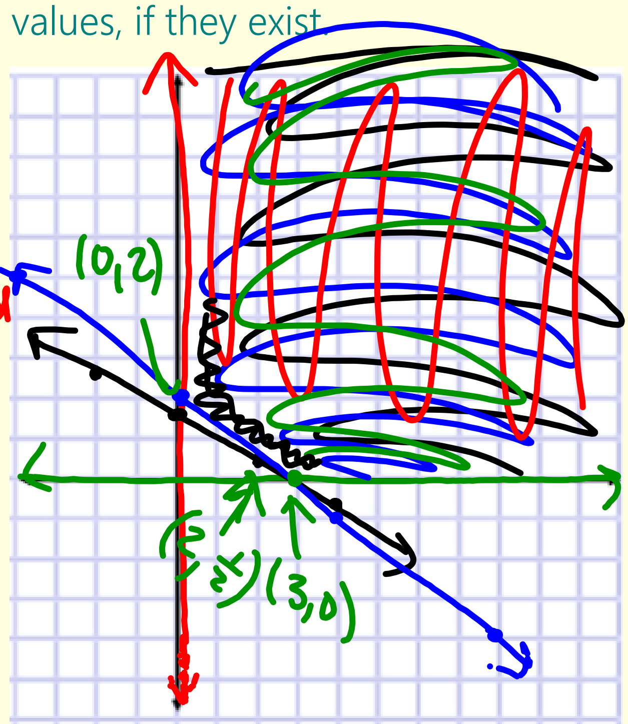
B. Find the maximum and minimum values, if they exist.

Constraints:

$$\begin{cases} x + 2y \ge 3 \\ 3x + 4y \ge 8 \\ x \ge 0 \\ y \ge 0 \end{cases}$$

$x \ge 0$ vertical solid line

$y \ge 0$ horizontal solid line



$$x + 2y \ge 3$$
$$-x \qquad -x$$
$$\frac{2y}{2} \ge \frac{3}{2} - \frac{x}{2}$$
$$y \ge \frac{3}{2} - \frac{1}{2}x$$
$$m = \frac{-1}{2} \quad b = 1\frac{1}{2}$$

$$3x + 4y \ge 8$$
$$-3x \qquad -3x$$
$$\frac{4y}{4} \ge \frac{8}{4} - \frac{3x}{4}$$
$$y \ge 2 - \frac{3}{4}x \qquad m = \frac{-3}{4} \quad b = 2$$

$E = x + y$

$(0,2)$	$(2, \frac{1}{2})$	$(3,0)$
$E = 0 + 2$	$E = \frac{2 \cdot 2}{1 \cdot 2} + \frac{1}{2}$	$E = 3 + 0$
$\boxed{E = 2 \text{ min}}$	$E = \frac{4}{2} + \frac{1}{2}$	$E = 3$
	$E = \frac{5}{2}$ or $2\frac{1}{2}$	

$\boxed{\text{no max}}$

since the graph is unbounded.

EXAMPLE 3

objective function

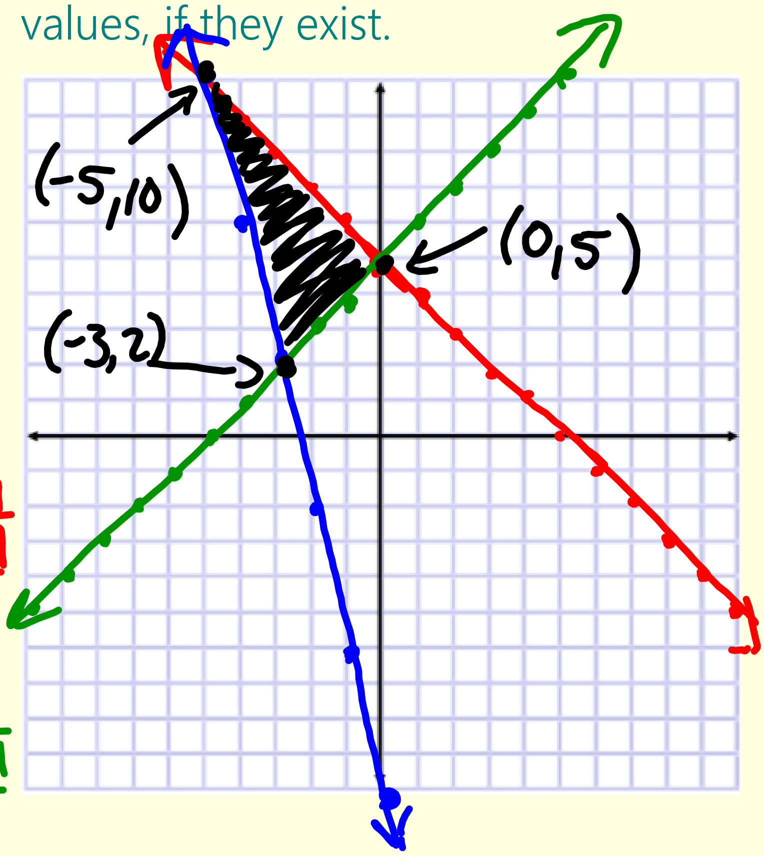
M = 3x + 2y

Constraints:

$$\begin{cases} x + y \le 5 \\ y - x \ge 5 \\ 4x + y \ge -10 \end{cases}$$

A. Graph the feasible region.

B. Find the maximum and minimum values, if they exist.



$x + y \le 5$
$-x \quad -x$
$y \le 5 - x$ $\quad m = \frac{-1}{1}$ $\quad b = 5$

$y - x \ge 5$
$+x \quad +x$
$y \ge 5 + x$ $\quad m = \frac{1}{1}$ $\quad b = 5$

$4x + y \ge -10$
$-4x \quad -4x$
$y \ge -10 - 4x$ $\quad m = \frac{-4}{1}$ $\quad b = -10$

$M = 3x + 2y$

(-5,10)	(-3,2)	(0,5)
$M = 3(-5) + 2(10)$	$M = 3(-3) + 2(2)$	$M = 3(0) + 2(5)$
$= -15 + 20$	$= -9 + 4$	$= 0 + 10$
$M = 5$	$\boxed{M = -5 \text{ min}}$	$\boxed{M = 10 \text{ max}}$