11.2 Part 1 Arithmetic Sequence

Sequence -> A set of numbers in a specific order

Term -> each number in a sequence

Terms are usually symbolized using a lower case number. For example, the first term will be written as a_1, the second term will be a_2, etc.

When each term in a sequence can be found by adding the same number to the previous term, that is an ARITHMETIC SEQUENCE.

Arithmetic Sequence -> a sequence in which each term, after the first, is found by adding a constant (called the common difference) to the previous term

The common difference is symbolized by the variable d.

To find the next terms in an arithmetic sequence, first find the common difference (d) by subtracting any term from its succeeding term.

Then, add the common difference to the last term to find the next few terms.

Example: Find the next FOUR terms of the arithmetic sequence:

$$\begin{array}{r} 39 \\ -\,33 \\ \hline 6 \end{array}$$

33, 39, 45, 51, 57, 63, 69, ...

+6 +6 +6 +6 +6 +6

Example: Find the next FOUR terms of the arithmetic sequence:

$$\begin{array}{r} 21 \\ -26 \\ \hline -5 \end{array}$$

26, 21, 16, 11, 6, 1, -4...

-5 -5 -5 -5 -5 -5

A formula to any term of an arithmetic sequence can be found if you know the first term and the common difference.

This type of formula is known as the recursive formula.

Recursive -> each succeeding term is formulated from one or more previous terms

Formula for the n^{th} Term of an Arithmetic Sequence:

The n^{th} term, a_n, of an arithmetic sequence with the first term, a_1, and common difference, d, is given by

$$a_n = a_1 + (n - 1)d$$

Example: Find the n^{th} term of each arithmetic sequence.

$a_1 = -1$ $d = -10$ $n = 25$

$$a_n = a_1 + (n-1)d$$

$$a_{25} = -1 + (25-1)(-10)$$

$$= -1 + (24)(-10)$$

$$= -1 + -240$$

$$\boxed{a_{25} = -241}$$

Example: Find the n^{th} term of each arithmetic sequence.

$a_1 = 2 \qquad d = \frac{1}{2} \qquad n = 8$

$$a_n = a_1 + (n-1)d$$
$$= 2 + (8-1)\left(\tfrac{1}{2}\right)$$
$$= 2 + (7)\left(\tfrac{1}{2}\right)$$
$$= \frac{2 \cdot 2}{1 \cdot 2} + \frac{7}{2} = \frac{4}{2} + \frac{7}{2}$$
$$\boxed{a_8 = \frac{11}{2}}$$

Example: Complete each statement.

a_n $\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ a_1

124 is the __19__th term of −2, 5, 12, ...

$+7 \quad d = 7$

$$a_n = a_1 + (n-1)d$$
$$124 = -2 + (n-1)(7)$$
$$124 = -2 + 7n - 7$$
$$124 = -9 + 7n$$
$$+9 \qquad +9$$
$$\frac{133}{7} = \frac{7n}{7}$$
$$n = 19$$

Example: Complete each statement.

a_n a_1

-28 is the ___8___ th term of 7, 2, -3, ...

-5 $d=-5$

$$a_n = a_1 + (n-1)d$$
$$-28 = 7 + (n-1)(-5)$$
$$-28 = 7 + -5n + 5$$
$$-28 = 12 + -5n$$
$$-12 \quad -12$$
$$\frac{-40}{-5} = \frac{-5n}{-5}$$
$$n = 8$$

Example: Find the indicated term in each arithmetic sequence.

a_1

a_{10} for 8, 3, -2, ...

$n=10$ -5 $d=-5$

$$a_n = a_1 + (n-1)d$$
$$a_{10} = 8 + (10-1)(-5)$$
$$= 8 + (9)(-5) = 8 + -45$$
$$\boxed{a_{10} = -37}$$

Sometimes, you may know two terms of a sequence that are not in consecutive order.

The terms between any two nonconsecutive terms of an arithmetic sequence are called ARITHMETIC MEANS.

Use the n^{th} term formula to find the common difference. Then, use the common difference to find the arithmetic means.

Example: Find the missing terms in each arithmetic sequence.

$a_1=2$ $a_n=20$

2, 5, 8, 11, 14, 17, 20

+3 +3 +3 +3 +3 +3

$n=7$

$a_n=a_1+(n-1)d$

$20=2+(7-1)d$

$20=2+6d$

$-2 \quad -2$

$\frac{18}{6}=\frac{6d}{6}$

$d=3$

Example: Find the missing terms in each arithmetic sequence.

$\underline{56}, \overset{a_1}{49}, \underline{42}, \underline{35}, \overset{a_n}{28}$

$+7 \quad -7 \quad n = 4 \quad -7$

$$a_n = a_1 + (n-1)d$$
$$28 = 49 + (4-1)d$$
$$28 = 49 + 3d$$
$$-49 \quad\quad -49$$
$$\frac{-21}{3} = \frac{3d}{3}$$
$$d = -7$$