Fundamental Theorem of Algebra

A polynomial of degree n has exactly n roots (zeros) in the set of complex numbers.

Roots or zeros may be rational (integers or fractions), irrational (square roots), or imaginary (i).

II. Find ALL Zeros

STEPS: $\pm \frac{p}{q}$

1. List all possible roots.
2. Test each possibility until you find one zero.
3. Divide by the zero (using synthetic division) to get depressed polynomial.
4. Repeat steps 1 to 3 until the depressed polynomial is a quadratic. exponent of 2
5. Solve the quadratic by factoring, square roots, or the quadratic formula to get the last 2 zeros.

EXAMPLES: Find all the zeros.

4. $f(x) = 1x^3 - 5x^2 + 3x + 9$ (q = 1, p = 9)

p: 1 3 9

q: 1

$\pm\frac{p}{q}$: ± 1 ± 3 ± 9

	1	−5	3	9
1	+ 1	−4	−1	9 ≠ 0

	1	−5	3	9
x = 3	+ 3	−6	−9	0 = 0 ✓
	$1x^2$	−2x	−3	

$(x-3)(1x^2-2x-3)$

Sum −2	prod. −3
−3 + 1	−3 · 1

$(x-3)(1x-3)(1x+1)$

$\frac{-3}{1}$ $\frac{1}{1}$

$x-3=0$ $x-3=0$ $x+1=0$

$+3\ +3$ $+3\ +3$ $-1\ -1$

$x=3$ $x=3$ $x=-1$

EXAMPLES: Find all the zeros.

5. $f(x) = 1x^3 + 3x^2 - 4$ (q = 1, p = 4)

p: 1 2 4

q: 1

$\pm\frac{p}{q}$: ± 1 ± 2 ± 4

−4	1	3	0	−4
	+	−4	4	−16
	1	−1	4	−20 ≠ 0

$x=1$	1	3	0	−4
	+	1	4	4
	$1x^2$	$4x$	4	0 = 0 ✓

$(x-1)(1x^2+4x+4)$ Sum 4 | Prod 4

$2+2$ | $2\cdot 2$

$(x-1)(1x+2)(1x+2)$ $\frac{2}{1}$ $\frac{2}{1}$

$x-1=0$ $x+2=0$ $x+2=0$

$+1\ +1$ $-2\ -2$ $-2\ -2$

$x=1$ $x=-2$ $x=-2$

EXAMPLES: Find all the zeros.

7. $1x^3 - 5x^2 + 11x - 10 = 0$

p: 1, 2, 5, 10

q: 1

$\pm\frac{p}{q}$: ± 1, ± 2, ± 5, ± 10

-5 | 1, −5, 11, −10

+ ↓, −5, 50, −305

1, −10, 61, −315 $\neq 0$

$x=2$ | 1, −5, 11, −10

+ ↓, 2, −6, 10

$1x^2$, $-3x$, 5, $0 = 0$ ✓

$(x-2)(1x^2-3x+5)$ Sum 3 | prod 5

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$$x=\frac{-(-3)\pm\sqrt{(-3)^2-4(1)(5)}}{2(1)}$$

$$x=\frac{3\pm\sqrt{9-20}}{2}=\frac{3\pm\sqrt{-11}}{2}$$

$x-2=0$

$x=2$

$x=\frac{3\pm i\sqrt{11}}{2}$