



Example 5

$x = -2$

Let  $P(x) = 2x^3 - 3x^2 - 11x + 6$ . Show that  $P(-2) = 0$ , and use this fact to write  $P(x)$  in factored form.

Then find the zeros.

$$\begin{aligned} \textcircled{1} P(-2) &= 2(-2)^3 - 3(-2)^2 - 11(-2) + 6 \\ &= 2(-8) - 3(4) + 22 + 6 \\ &= -16 - 12 + 22 + 6 = -28 + 22 + 6 = -6 + 6 = 0 \end{aligned}$$

$$\begin{array}{r} \textcircled{2} \quad x = -2 \quad \Big| \quad 2 \quad -3 \quad -11 \quad 6 \\ \quad \quad \quad \quad \quad \quad \quad \downarrow \quad -4 \quad 14 \quad -6 \\ \hline \quad \quad \quad 2x^2 - 7x \quad 3 \quad \boxed{0} \end{array}$$

remainder  $\swarrow$

$$P(x) = (x+2)(2x^2 - 7x + 3)$$

sum -7	prod. 6
-6 + -1	-6 · -1

$$P(x) = (x+2)(x-3)(2x-1)$$

$$\frac{-3}{1} = \frac{-6}{2} \quad \frac{-1}{2}$$

$$(x-3)(2x-1)$$

$$\begin{array}{ccc} \textcircled{3} \quad x+2=0 & x-3=0 & 2x-1=0 \\ \quad \quad \quad \begin{array}{cc} -2 & -2 \end{array} & \quad \quad \begin{array}{cc} +3 & +3 \end{array} & \quad \quad \begin{array}{cc} +1 & +1 \end{array} \\ \quad \quad \quad \boxed{x=-2} & \quad \quad \boxed{x=3} & \quad \quad \begin{array}{c} 2x=1 \\ \boxed{x=\frac{1}{2}} \end{array} \end{array}$$

Example 6

Let  $P(x) = x^4 - 6x^3 + 3x^2 + 26x - 24$ . If  $P(3) = 0$  and  $P(-2) = 0$ , write  $P(x)$  in factored form.

$x=3$

$x=-2$

$$\begin{array}{r} 3 \overline{) 1 \quad -6 \quad 3 \quad 26 \quad -24} \\ + \quad \downarrow \quad 3 \quad -9 \quad -18 \quad 24 \\ \hline 1x^3 - 3x^2 - 6x \quad 8 \quad 0 \end{array}$$

$$\begin{array}{r} -2 \overline{) 1 \quad -3 \quad -6 \quad 8} \\ + \quad \downarrow \quad -2 \quad 10 \quad -8 \\ \hline 1x^2 - 5x \quad 4 \quad 0 \end{array}$$

$$P(x) = (x-3)(x+2)(x^2-5x+4)$$

$$P(x) = (x-3)(x+2)(x-1)(x-4)$$

$$\begin{array}{r|l} \text{sum } -5 & \text{prod. } 4 \\ \hline -1 + -4 & -1 \cdot -4 \\ \frac{-1}{1} & \frac{-4}{1} \\ & (x-1)(x-4) \end{array}$$

Example 7

Given the polynomial  $2x^3 + x^2 - 13x + 6$  and the factor  $x + 3$  find the remaining factors and write the polynomial in factored form. Find the zeros.

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline x=-3 \end{array}$$

$$\begin{array}{r} -3 \overline{) 2 \quad 1 \quad -13 \quad 6} \\ \quad + \downarrow \quad -6 \quad 15 \quad -6 \\ \hline \quad \quad 2x^2 - 5x \quad 2 \quad 0 \end{array}$$

$$P(x) = (x+3)(2x^2 - 5x + 2)$$

$$P(x) = (x+3)(x-2)(2x-1)$$

$$\begin{array}{c|c} \text{sum } -5 & \text{prod. } 4 \\ \hline -4 + -1 & -4 \cdot -1 \end{array}$$

$$-\frac{2}{1} = \frac{-4}{2} \quad -\frac{1}{2}$$

$$(x-2)(2x-1)$$

$$\begin{array}{r} x+3=0 \\ -3 \quad -3 \\ \hline \end{array} \quad \begin{array}{r} x-2=0 \\ +2 \quad +2 \\ \hline \end{array} \quad \begin{array}{r} 2x-1=0 \\ +1 \quad +1 \\ \hline \end{array}$$

$$x = -3$$

$$x = 2$$

$$\begin{array}{r} 2x = \frac{1}{2} \\ \frac{2}{2} \\ \hline x = \frac{1}{2} \end{array}$$

Example 8

Given the polynomial  $4x^3 + 13x^2 - 37x - 10$  and the factor  $x - 2$ , find the remaining factors and write the polynomial in factored form. Find the zeros.